MODELING RISKS IN INFRASTRUCTURE ASSET MANAGEMENT

A Dissertation

by

SEYED REZA SEYEDOLSHOHADAIE

Submitted to the Office of Graduate Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

August 2011

Major Subject: Industrial Engineering

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ABSTRACT

Modeling Risks in Infrastructure Asset Management. (August 2011)

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The goal of this dissertation research was to model risk in delivery, operation and maintenance phases of infrastructure asset management. More specifically, the two main objectives of this research were to quantify and measure financial risk in privatizing and operational risks in maintenance and rehabilitation of infrastructure facilities. To this end, a valuation procedure for valuing large-scale risky projects is proposed. This valuation approach is based on mean-risk portfolio optimization in which a risk-averse decision-maker seeks to maximize the expected return subject to downside risk. We showed that, in complete markets, the value obtained from this approach is equal to the value obtained from the standard option pricing approach. Furthermore, we introduced Coherent Valuation Procedure (CVP) for valuing risky projects in partially complete markets. This approach leads to a lower degree of subjectivity as it only requires one parameter to incorporate user's risk preferences. Compared to the traditional discounted cash flow analysis, CVP displays a reasonable degree of sensitivity to the discount rate, since only the risk-free rate is used to discount future cash flows. The application of this procedure on valuing a transportation

public-private partnership is presented.

Secondly, a risk-based framework for prescribing optimal risk-based maintenance and rehabilitation (M&R) policies for transportation infrastructure is presented. These policies guarantee a certain performance level across the network under a predefined level of risk. The long-term model is formulated in the Markov Decision Process framework with risk-averse actions and transitional probabilities describing the uncertainty in the deterioration process. Conditional Value at Risk (CVaR) is used as the measure of risk. The steady-state risk-averse M&R policies are modeled assuming no budget restriction. To address the short-term resource allocation problem, two linear programming models are presented to generate network-level polices with different objectives. In the first model, decision-maker minimizes the total risk across the network, and in the second model, the highest risk to the network performance is minimized.

To life

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CHAPTER I

INTRODUCTION

A. Motivation

Availability of civil infrastructure systems such as transportation, power, water and communications is the driving force behind the economic development of any region. Infrastructure has a direct effect on the quality of life and overall well being of the society. The massive networks of civil infrastructure systems built to support and maintain the economic growth in countries around the world are the result of decades of planning and development, making them the largest asset in public ownership. Developing and maintaining infrastructure networks requires a long-term plan and allocation of a significant amount of resources.

Public, semi-public or private agencies are responsible for building, maintaining or improving infrastructure systems in a sustainable and effective way. In this effort, infrastructure asset management models are used to incorporate an economic assessment of trade-offs among alternative investments for construction, maintenance and rehabilitation of such facilities. By using these models, a network of infrastructure facilities can be managed in a cost-effective, efficient and reliable manner. In many countries around the world, infrastructure systems are built and maintained by federal, state and local governments, however, there is an interest in attracting private funding to develop or maintain infrastructure facilities.

In the United States (U.S.), infrastructure systems have traditionally been owned and operated by government agencies. However, as the result of tax reduction in recent decades, a lower level of funding is available for infrastructure spending. The

This dissertation follows the style of Mathematical Programming Journal.

unavailability of resources in underfunded agencies responsible for maintaining infrastructure facilities often results in deferred maintenance on facilities that must undergo periodic maintenance to maximize their service lives. Infrastructure facilities deteriorate more rapidly due to the lack of proper maintenance and they will become exposed to the risk of severe structural damages. Consequently, the overall cost of maintenance increases significantly as more expensive actions will be required to restore these facilities to acceptable condition levels.

Transportation infrastructure systems, in particular, are more visible to users and their deficiencies have apparent impact on society. According to the U.S. Department of Transportation, in the U.S., 74% percent of the $8.4 trillion worth of commodities is transported by trucks on interstate highways. Despite the massive network of highways, the cost of traffic congestion in terms of loss of productivity and fuel costs in the U.S. has been estimated to be $115 billion in 2010, causing, on average, every urban resident to spend an extra 34 hours of travel time and use 28 gallons of fuel [55]. In 2000, the total expenditure by all levels of government on transportation infrastructure was $64.6 billion. However, the Federal Highway Administration (FHWA) estimates that the spending by all levels of government would have to increase by 17.5% to reach its projected $75.9 billion needed to maintain the current condition levels, and by 65.3% to reach $106.9 billion needed to improve the conditions of roads and highways [3].

B. Goals and Objectives

The goal of this dissertation research is to model risk in delivery, operation and maintenance phases of infrastructure asset management. More specifically, the two objectives of this dissertation are:

1. Develop a valuation approach for large-scale engineering projects.

2. Develop a framework for risk-based maintenance and rehabilitation of transportation infrastructure.

A natural way to increase funding for infrastructure spending for construction of new and maintenance of the existing facilities is by attracting private investment for such projects. Many states in the U.S. have enacted legislation that allows private sector investment in infrastructure projects through Public-Private Partnerships (PPPs). PPPs are contractual agreements in which the private sector gets involved in part, or in the entire, process of designing, financing, constructing and operating public infrastructure facilities [7].

An important part of privatizing infrastructure assets is project valuation as both parties embark on quantifying the risks and finding the value of investment. Similar to any other risky assets, the value of an infrastructure project is proportional to its underlying risks and prospective return. Infrastructure projects are usually capital intensive and require significant investment, but at the same time they cannot be traded in open markets. The large-scale and non-traded nature of infrastructure projects require special consideration in valuing such assets. One of the challenges arising in the valuing risky assets stems from uncertainties that are unique to the project. The first objective is to develop a valuation procedure for valuing large-scale engineering projects.

The second objective is aimed at identifying and measuring risk stemming from the uncertain deterioration of transportation infrastructure facilities. The goal is to develop a risk-based framework for prescribing long-and short- term resource allocation decisions for maintenance or rehabilitation of a network of transportation infrastructure facilities.

Transportation infrastructure is being used for demonstrating the models developed in this dissertation research since, traditionally, transportation infrastructure has been used for proof of concept, however, these models can be implemented on other classes of infrastructure.

C. Organization

This dissertation is organized as follows: Chapter II provides background and definitions. Chapter III, discusses risk in general, the concept of risk management and ways to quantify risk from a perspective of a risk-averse decision-maker. Chapter IV presents a procedure for valuing large-scale engineering projects. Chapter V presents a framework for risk-based maintenance and rehabilitation of a network of transportation infrastructure facilities. The summary of findings, conclusions and future research directions are discussed in Chapter VI.

CHAPTER II

BACKGROUND

A. Infrastructure Asset Management

Several definitions for infrastructure asset management have been proposed. According to FHWA [5],

> "Asset management is a business process and a decision-making framework that covers an extended time horizon, draws from economics as well as engineering, and considers a broad range of assets. The asset management approach incorporates the economic assessment of trade-offs among alternative investment options and uses this information to help make cost-effective investment decisions."

According to the Organization for European Cooperation and Development Working Group [63],

> "Asset management goes beyond the traditional management practice of examining singular systems within road networks; i.e., pavements, bridges, etc.; and looks at the universal system of a network of roads and all of its components to allow comprehensive management of limited resources."

In the broadest sense, infrastructure management spans all stages of infrastructure life cycle: planning, programming, budgeting, design, construction, and operation and maintenance [27]. The objective of asset management is to build and operate infrastructure facilities in the most reliable, cost effective and sustainable way.

Infrastructure management can also be viewed from two managerial levels: network and project [6, 47]. At the network level, agencies manage all elements of a single infrastructure facility or a network of facilities. In some cases, this may involve grouping facilities together to form distinct subsets. The purpose of the network-level management process is normally related to planning, programming, and determining funding levels, prioritizing needs and analyzing the impact of various funding scenarios on the future condition of the infrastructure system and the overall welfare of the community. One of the challenges in this process is measuring the benefits of alternative actions. The benefits of a public project are not easy to quantify due to the distributed nature of the benefits and also non-uniform utility function of different components of infrastructure users. In addition, unlike the private sector, public agencies are not-for-profit organizations.

At a project level, however, only a single facility or a portion of a facility that in some cases corresponds to an original construction project is considered. The purpose of the project level is to determine the best strategy possible for a particular facility. The primary results of project-level management include an analysis of the deterioration process; identification of possible maintenance strategies; and selection of the best strategy, given resource limitations [47].

B. Asset Ownership

Depending on the legal and political structure governing the provision of infrastructure, facilities can be owned and operated by public, semi-public or private agencies. In many states or countries, the private sector plays an active role in different aspects of infrastructure asset management.

Public-private Partnerships (PPPs) or Private Finance Initiatives (PFIs) are provisions under which the private sector gets involved in the process of designing, financing, constructing and operating public infrastructure facilities. Like any other partnership, PPPs share responsibilities by both public and private parties.

While private sector involvement in managing infrastructure assets is relatively new in the U.S., Western European countries like France, Italy, Spain and England have extensive experience with PPPs. Across the U.S. an increasing interest in adopting PPPs is evident. As of 2006, 21 states had enacted or introduced PPP legislation, allowing the privatization of existing assets and many more states have such legislation in progress [1].

Figure 1 depicts the public versus private financing of infrastructure projects. In the public finance provision, government is directly involved in operating, financing or construction of the project. On the other hand, under private financing, a privately held entity called a *Special Purpose Vehicle* is created and enters into a long-term contract with the government to provide financing and/or operation or construction for an infrastructure project.

1. PPPs in Transportation Infrastructure

PPPs have been used in public transportation projects since the 1960's. Spain and France were the first European countries to use them in their modern form to develop their highway networks. England's efforts to privatize its public transport services goes back to the country's economic problems of the 1970's, after government investment in transportation projects reduced significantly due to international oil crisis of 1973. The experiences gained in these projects led to the introduction of Public Finance Initiative (PFI) in 1992, which is the prototype model for modern PPPs [4].

Using PPPs for transportation projects has gained much interest in the U.S. in recent years. Transportation infrastructure PPPs can take different forms. Typical forms defined by FHWA [2] are Design-Build, Operation and Maintenance (O&M) Concession, Design-Build-Operate, Long Term Lease and Lease-Develop-Operate.

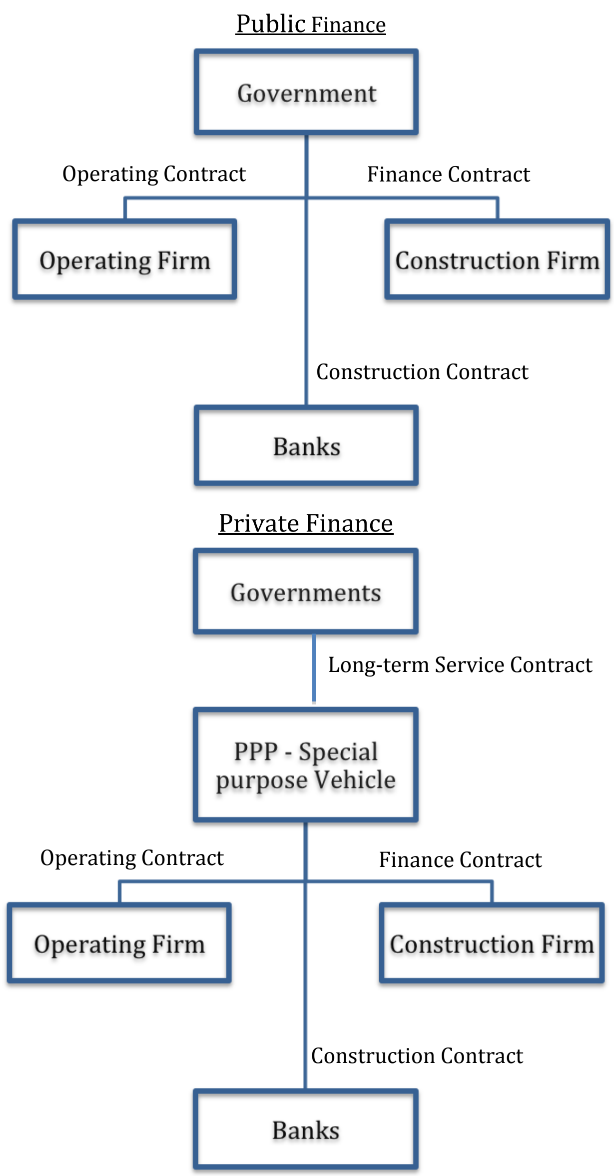


Fig. 1. Public vs. private finance.

- **Design-Build.** In the Design-Build model, a single entity or a joint venture enters into agreement for a single contract that involves both engineering services and construction. Example: Design and construction of 47-mile E-470 toll road in Colorado.

- **O&M Concession.** Operation and Maintenance (O&M) agreements allow private entities to operate and manage existing facilities. Contracts can be based on a fixed fee for O&M or based on delivering a certain level of service. Example: the five year concession for maintenance of Washington D.C. streets, tunnels, pavements, bridges, etc.

- **Design-Build-Operate (Maintain).** In this model, engineering/architecture services and construction, along with maintenance of the facility, are integrated into a single partnership and the project is financed by the public entity. Example: The Hudson-Bergen Light Rail transit system in New Jersey.

- **Design-Build-Finance-Operate.** This is the same as Design Build Operate with the exception that the private entity, which can be a single entity, or a consortium or a joint venture, also has the responsibility to finance the project. Example: the I-495 Capital Beltway high occupancy toll (HOT) lane in Fairfax, Virginia.

- **Long Term Lease.** In this model, an existing facility (*e.g.* highway) is leased to a private entity on a long term basis, conferring the right to collect tolls. Examples: the 99-year lease of the Chicago Skyway.

- **Lease-Develop-Operate.** In the lease, develop and operate model, a long-term lease is signed to grant a private party the right to operate an existing facility. The private party can improve/expand the facility and, at the end of the lease period, redeem the investment principal and the returns. Example: the 99-year agreement to enhance, manage, operate, maintain and collect tolls on the Pocahontas Parkway in Richmond, Virginia.

2. Benefits of PPPs

In general, involving the private sector in infrastructure asset management can provide a diverse set of benefits. Akintoye et al. [7] described these benefits as follows:

- *Enhance government's capacity to develop an integrated solution:* PPPs give a government the opportunity to implement an integrated and better set of services to the public, which it would otherwise be unable to implement, due to budget limitations.

- *Facilitate creative innovation approaches:* The underlying project attracts various bidders to compete, based on their creative approaches to deliver required outputs.

- *Reduce the cost to implement:* PPPs allow the same level of service to be delivered with lower cost in design, construction and operational period.

- *Reduce the time to implement the project:* Traditionally, infrastructure projects are broken down into pieces for which each part is incorporated into a multi-year plan. PPPs enable planning new phases while construction is underway, facilitating on-time completion of the entire project. This would have been impossible without private investment due to budget restrictions.

- *Transfer certain risks to the private partner:* An effective transfer of risk allows better management of risks since each party in the partnership will take appropriate risks according to its ability to manage that type of risk.

- *Attract larger, potentially more sophisticated bidders to the project:* PPPs enable the government to attract different types of bidders and increase competition among bidders.

- *Access skills, experience and technology:* The government can gain access to new technology and intellectual property developed in the private sector as a result of PPP projects.

Despite these benefits, PPPs have also several drawbacks such as the possibility of service disruption due to financial distress of the private party, the possibility of lengthy and costly contract negotiations and increased cost of construction. Maybe the most important drawback of PPPs is the loss of control over public services for a long period of time. Risk management and mitigation can play an important role in PPPs. Sound risk-management practices can significantly mitigate different types of risk in PPPs; however, they cannot be totally eliminated.

3. Risk Allocation and Management

As one of the goals of PPPs is to share project risk between public and private sectors, it is very important that each party undertakes the types of risk that it can best manage. Specific risks associated with PPPs create various known or unknown liabilities for governments. According to Shwartz et al. [56], project related risk encountered in PPPs can be categorized and described as:

- Construction Risk: Risks stemming from low quality of design and construction as well as project overruns.

- Financial Risk: Project's inability to generate enough cash flow to repay loans or capital investment.

- Demand Risk: The possibility that the demand for the project reduces due to economic slowdown or other issues and/or there is not enough demand for using the service provided by the facility.

- Availability Risk: When facility does not provide the required services due to low quality of design and construction or when service is frequently disrupted, for example, by maintenance needs.

- Force Majeure: Risks that cannot be controlled by private or public entities, for example, natural disasters.

- Residual Value Risk: Uncertainty over the market price of the asset at the time when the contract expires, as in many situations an infrastructure facility will be returned to the public owner after a certain period of time.

Experiences of different countries in privatizing infrastructure suggest that economic infrastructure projects, such as transportation infrastructure, are good candidates for PPPs. Transportation infrastructure, such as roads, railways and ports, is more attractive to the private sector than other infrastructure services, such as health-care and education, since their risks and returns are easier to quantify and they usually offer comparatively higher rates of return. It is also easier to charge users directly for such infrastructure projects [8].

The major decision in the development of PPPs for both public and private partners is whether to proceed with the project or not. This decision can be made by accurately valuing the project. Financial risks and demand risk arise when project revenues are not accurately estimated and the investment is not justified based on the realistic assumptions of demand and construction or operating costs. Lack of economic justification for the project will eventually lead to service disruptions, creating otherwise non-existing financial and institutional burden for the government. On the other hand, when demand is underestimated or construction costs are overestimated, tax payers are overcharged for the service provided. This is even more important when the government is providing subsidies to supplement revenue in order to make the project financially attractive to private investors. Therefore, it is critical for both public and private entities to manage financial risk through sound valuation approaches.

4. Real Options in Infrastructure Projects

A real option is an alternative, but not an obligation, to invest in a risky project. Real options relate to a wide range of business decisions, like investing in a new project, expanding product offerings, or designing a supply chain. In PPPs, real options can be offered by the public entity to reduce investment risks and make the project attractive for private investors. In general, a real option provides the opportunity for the decision maker to implement an action after specific uncertainties are resolved.

Two common real options for infrastructure projects involve deferring investment and abandoning the project.

- Option to defer investment: This option is similar to buying a “call” option on a stock, which gives an investor the ability to postpone investment until uncertainty is resolved. If a real option is acquired by an investor, an investment decision can be made over a range of time until the opportunity cost of investing in the project outweighs other available investment options.

- Option to abandon the project: This option is similar to buying a “put” option on a stock, which gives a decision-maker flexibility to abandon the project if the proceeds from the investment do not justify its cost. This option can be very attractive to investors as it allows them to sell the project at a reasonable price and recover the initial investment.

The Balck-Scholes formula [10], developed for valuing options on publicly traded assets, can be applied to value real options only if there exists a twin security or a portfolio that can replicate the cash flows from real option. Refer to [61] for a complete discussion of real options.

C. Maintenance and Rehabilitation (M&R) of Infrastructure Facilities

All infrastructure facilities deteriorate over time and must undergo periodic maintenance to attain their maximum service lives. Although maintenance can slow the deterioration rate of infrastructure facilities, deterioration must eventually be reversed by actions such as rehabilitation, reconstruction or retrofit so that the facility can remain operational.

Preventive maintenance includes nonstructural maintenance actions that are intended to prolong the life of the facility and are applied during the life of the facility. For example, for pavement segments, actions like slurry sealing, or applying a thin surface overlay are considered maintenance actions, because they reduce the rate of deterioration, while a thick surface overlay is considered rehabilitation because it enhances structural strength.

In some cases, due to lack of effective maintenance planning, a major rehabilitation or reconstruction is needed. The cost of rehabilitation for severely deteriorated facilities is sometimes as much as seven times more expensive than the cost of periodic preventive maintenance actions [20]. Figure 2 shows how the cost increases as a result of delayed maintenance. The curve indicated by “A” depicts the facility condition under preventive maintenance while curve B shows the deterioration process with no maintenance action. For a pavement with a 25-year life span, 40-percent of the drop in quality occurs during the first 75% of the life of the facility; after that, deterioration accelerates and the next 40-percent drop in quality occurs during only 12% of the life.

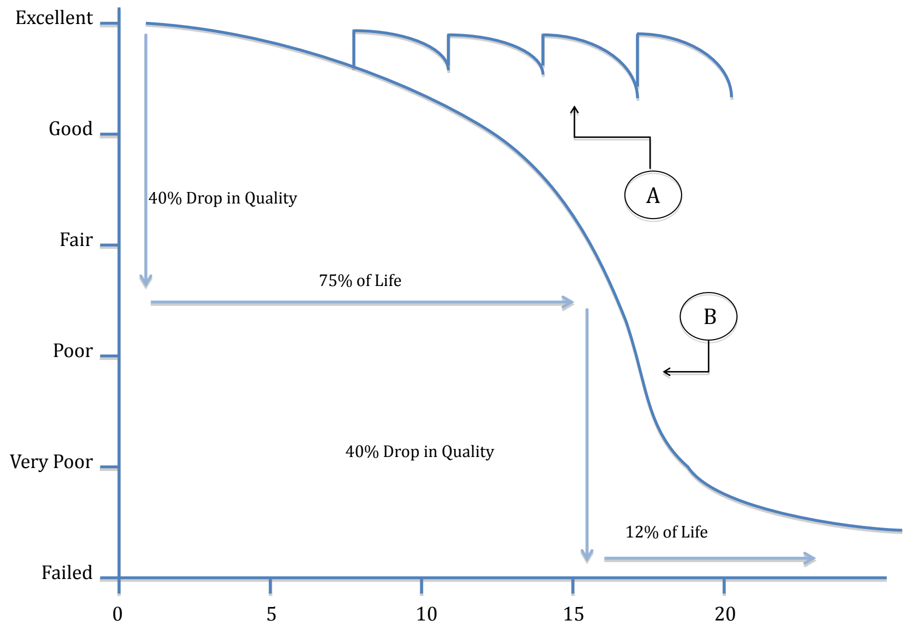


Fig. 2. Cost of delayed maintenance.

1. Network-level Management System

Many public transportation agencies have designed and implemented pavement management systems for collecting information and prescribing timely and cost-effective M&R actions to optimally allocate resources. In transportation infrastructure terminology, a facility is a segment or piece of road or highway within a network of roads or highways. Pavement management tools were among the first infrastructure asset management decision support softwares, but others soon followed [27]. Pavement management systems have often been used to illustrate the concept, as pavement deterioration models and treatment actions are more studied than others. According to the American Association of State Highway and Transportation Officials (AASHTO) Guidelines for Pavement Management Systems [6], a network-level pavement management system comprises six main elements: 1) an inventory; 2) condition assessment; 3) determination of fund needs; 4) identification and prioritization of candidate projects when funds are constrained; 5) a method to determine the impact of funding decisions on future condition and funds needed; and 6) a feedback process. The following paragraphs describe each element in more detail.

Inventory. Inventory includes information about size, location, number of lanes, functional classification and all the basic information concerning each facility that is a road segment within a network of roads.

Condition assessment. Condition assessment depends on the type of facility and starts with gathering data about the severity of distress and overall structural integrity of the facility. This assessment allows the degree of deterioration to be determined so that feasible maintenance and rehabilitation actions for each segment and costs of maintenance and repair can be estimated. This information can be summarized by an index, *e.g.*, International Roughness Index (IRI), a condition assessment, which represents the pavement roughness or Pavement Condition Index (PCI), which is determined by a visual inspection that identifies the types, severities and quantities of distresses.

Determination of fund needs. Once the inventory and condition data are collected, agencies can determine the type of actions needed along with the requested resources. Required actions are usually determined by comparing the forecasted future state of the facility with and without maintenance or rehabilitation actions and the effect these actions have on the facility in terms of a condition index at the end of a specific time frame.

Prioritization candidate sections when funds are constrained. Once an agency determines the funds needed to maintain the facility/network in the desired condition, funding requirements must be compared to fund availability [47]. If available funds are less than needed in any of the years, they must be allocated for the management sections over the planning horizon. Generally, the goal is to provide the greatest overall improvement in network condition for the funds expended.

Determining the impact of funding decisions on the future condition and fund needs. The goal of government agencies is to provide the maximum social benefit for the money provided to them by the public. However, other criteria can be used instead or in combination with social benefits for allocating funds.

Feedback process. Most infrastructure management decision support systems are implemented using empirical models and a limited amount of data. To enhance the quality and reliability of the system, a process is needed to provide feedback on the accuracy and reliability of prior estimates so that future estimates can be improved.

An important part of infrastructure asset management, as evidenced by network-level pavement management systems, is resource allocation through prioritization of candidate facilities at the network level. Since the deterioration of an infrastructure facility is not deterministic, (*i.e.*, the exact condition of the facility after a certain period of time cannot be determined with certainty), a stochastic resource allocation problem should be formulated and solved to prescribe an optimal maintenance action for each facility, given facility condition and available resources. Having the current condition of each facility; the conditional probabilities of future states, based on the current condition; and the cost of M&R actions, the minimum cost M&R action for any given facility with a specific condition can be modeled as a Markov decision process. The Markovian assumption implies that the condition of the facility at time $t + 1$ depends only on the condition at time t and the action applied to the facility at time t. The optimization model finds a minimum cost maintenance policy, namely an appropriate maintenance action for each possible condition state of the facility. However, at the network level with limited amount of resources, the optimal solution becomes computationally expensive to obtain because the number of state-space variables increases exponentially with the number of facilities in the network and the number of time periods in the planing horizon.

D. Summary

Infrastructure facilities are the driving force behind the economic development of a region. Infrastructure asset management models incorporate an economic assessment of tradeoffs among alternative investment decisions. Asset management can span all stages of the infrastructure life cycle: planning, programming, budgeting, design, construction, operation, and maintenance.

In this chapter, possible arrangements between private and public entities to own and operate infrastructure assets, the benefits of private sector involvement in asset management, and potential risks in privatizing these assets are described. More specifically, public-private partnerships, their benefits and common arrangements in transportation infrastructure are discussed.

While PPPs are gaining traction in terms of both the number of investments and the amount of capital invested, identifying and quantifying different types of risks involved in such partnerships remains a big challenge. One of the major issues in privatizing infrastructure projects through PPPs is project valuation. Financial and demand risks arise when project revenues are not accurately estimated and the investment is not justified based on the realistic assumptions of demand and construction or operating costs. The inability of private owners to manage project risks will eventually lead to service disruptions, creating otherwise non-existing financial and institutional burden for the government. Therefore, having a sound valuation approach is critical for both public and private entities for managing fiscal risks in PPPs.

Furthermore, in this chapter, issues related to maintenance and rehabilitation of infrastructure facilities are discussed. Infrastructure facilities should undergo periodic maintenance actions to maximize their service lives. Although preventive maintenance can slow the deterioration rate of an infrastructure facility, the effects of deterioration eventually must be reversed by conducting rehabilitation and reconstruction. In some cases, due to lack of effective maintenance planning, a major rehabilitation or reconstruction is needed under ideal timing. Many public transportation agencies have designed and implemented pavement management systems for collecting information and prescribing timely and cost-effective M&R actions to optimally allocate resources. However, finding optimal network-level M&R policies is challenging because of uncertain deterioration of infrastructure facilities and resource limitations.

CHAPTER III

MEASURING RISK

Risk stems from the uncertainty of the future outcomes and its effect on decision-maker's objectives. In other words, risk exists whenever there is a set of possible outcomes that can prevent the decision-maker from achieving his/her objectives. Based on the subject area, risk can be attributed to different unfavorable outcomes. In finance, risk could be the probability that the realized return is less than the expected return, while in an engineering setting, risk can be defined as the probability that entire or part of an engineering system does not meet expectations.

A. Risk Management

In a broad sense, managing risk involves identifying, quantifying and assessing different types of risk in the system along with strategies to manage them. A wide range of strategies, like accepting and preparing for the aftermath, reducing the impact, transferring the risk to other parties, and avoiding the risk all together, can be implemented to manage risk. Most recently, International Organization for Standardization (ISO) has published ISO 31000 [24], a family of standards relating to risk management practices. According to ISO 31000, the process of risk management consists of the following steps:

- Establishing the context
- Risk Assessment
  - Risk Identification
  - Risk Analysis

  - Risk Evaluation

- Risk Treatment

Establishing the context involves recognition of risks in the domain of interest, the stakeholder and all who will be affected by the risks. Risk assessment consists of identification, analysis, and evaluation of risks. Risk identification and analysis is a systematic approach to identifying and locating different types of risks that might exist along with the cause and effect of such events and their consequences for the system. It also involves estimating the probabilities of such events. The result of this step can be presented in a quantitative or qualitative manner depending on the type of the risk and the extent of information about the future events. The final step in risk assessment is risk evaluation, through which identified risks are evaluated and prioritized based on their level of importance. Risk treatment involves evaluation of risk treatment options with cost and benefit analysis and, ultimately, selection and implementation of the appropriate treatment option.

B. Quantifying Risk

The risk management standard that we discussed in the previous section contains two major elements. Firstly, after different types of risk are identified in the system, these risks must be expressed in either quantitative or qualitative form. Secondly, the decision-maker must allocate scarce resources to mitigate or eliminate the risks. Having limited resources at hand, often times resources should be allocated by formulating and solving an optimization model. Due to presence of random variables in the model, stochastic optimization techniques should be used to solve the problem which can be computationally expensive. As a result, using a measure of risk that can be incorporated into optimization modeling with less computational burden is important in choosing an appropriate measure of risk.

In general, two approaches for measuring risk are by measuring deviation of the underlying random variable from a constant value or by using a surrogate for the amount of risk (*e.g.*, mean or worst possible value of the future outcomes) [51]. Traditionally, *expected utility theory* and *certainty equivalent* have been used as a surrogate for measuring risk of risky bets. Expected utility theory is based on four axioms of rationality originally presented by Neumann and Morgenstern [44]. In the context of expected utility, certainty equivalent and *risk premium* are two important and closely related concepts. Certainty equivalent is defined as a payoff that a decision-maker considers as the equivalent of the risky bet or gamble and risk premium is the amount that a risk-averse decision-maker is willing to forgo to obtain a certain payoff. More generally, a decision-maker is said to be risk-neutral if the difference between expected payoff and certainty equivalent is zero; risk-averse, if the difference is positive; and risk-loving if the difference is negative.

1. Risk Aversion

Risk aversion is the tendency of a decision-maker to accept a certain amount instead of a gamble that offers equal or greater expected payoff. Suppose the decision-maker holds utility U for random variable X, which represents different realizations of wealth in the future. We denote by $E[U(X)]$ the expected utility of random variable X. The certainty equivalent is the amount y such that $U(y) = E[U(X)]$. $\mathrm{CE}[X]$ is a conventional notation for denoting the certainty equivalent of a random variable X.

Risk-averse utility functions take the form of non-increasing concave functions. Two major family of risk-averse utility functions are functions with the Constant Absolute Risk Aversion (CARA) property and functions with the Constant Relative Risk Aversion (CRRA) property. The exponential utility function $U(X) = -\exp(-aX)$, in which a is the coefficient of absolute risk aversion, exhibits the CARA property. This function has also Δ property which is an important property stating that, if a constant Δ is added to a random variable, the certainty equivalent is increased by the same amount,

$$CE(X + \Delta) = CE(X) + \Delta.$$

This property implies that the risk premium of the gamble does not depend on the initial wealth of the decision-maker.

Rabin [49] and Rabin and Thaler [50], have raised a question concerning the application of expected utility theory to explain risk-aversion for gambles involving both small and large stakes. They show that not accepting a small positive gamble, implies not accepting a large favorable gamble under the concave utility function. While utility theory plays an important role in the development of many theories in economics and decision-making under uncertainty, the difficulties and sometimes impracticalities in deriving a utility function and related parameters have hindered the application of expected utility in many practical settings.

Value at Risk. A risk measure that has been widely used in financial risk management practices and has been written in financial regulations (Basel I and Basel II) is Value at Risk (VaR). VaR, by definition, is the amount of loss at a certain confidence level. Given the random variable X with the cumulative distribution function $F_X(y) = P\{X \leq y\}$, and for some confidence level $\alpha \in (0, 1)$, Value at Risk with respect to α is,

$$\mathrm{VaR}_\alpha(X) = \min\{y | F_X(y) \geq \alpha\}.$$

Value at Risk is easy to understand and conveys an intuitive measure of risk. However, despite its ease of use and intuitiveness, VaR does not have appealing mathematical properties. $\mathrm{VaR}_\alpha(X)$ can increase significantly with a small change in α and is a non-convex and discontinuous function for discrete probability distributions, which makes it difficult to incorporate into optimization modeling. In addition, VaR only measures risk at certain level and ignores scenarios in the tail distribution. Chance constraint is equivalent to a VaR constraint in optimization and they are well known for introducing non-convexity in the problem.

A more modern definition of risk measures is presented by coherent measures of risk. Following the pioneering work of Artzner et al. [9] and Delbaen [15], in which they provided the basic axiomatic properties that a functional should have to be a good quantifier of risk, the properties of coherent risk measures are developed into four axioms of coherency as follows.

C. Coherent Measure of Risk

A coherent measure of risk is a functional $\mathcal{R} : \mathcal{L}^2 \to (-\infty, \infty]$ that has the following properties:

1. $\mathcal{R}(C) = C$ for all constants C.
2. $\mathcal{R}((1-\lambda)X + \lambda X') \leq (1-\lambda)\mathcal{R}(X) + \lambda\mathcal{R}(X')$ for $\lambda \in (0,1)$ (convexity).
3. $\mathcal{R}(X) \leq \mathcal{R}(X')$ when $X \leq X'$ (monotonicity).
4. $\mathcal{R}(X) \leq 0$ when $||X_k - X||_2 \to 0$ with $\mathcal{R}(X_k) \leq 0$ (closedness).

It will be a coherent measure of risk in basic sense if it also satisfies:

5. $\mathcal{R}(\lambda X) = \lambda\mathcal{R}(X)$ for $\lambda > 0$ (positive homogeneity).

In the original definition, the first property was presented as the following relation, which follows from (1) and (2) in the current definition,

$$\mathcal{R}(X + C) = \mathcal{R}(X) + C.$$

Also, the *subadditivity* property can be derived from (2) and (5),

$$\mathcal{R}(X + X') \leq \mathcal{R}(X) + \mathcal{R}(X'). \tag{3.1}$$

While coherent risk measures were developed initially for banking regulation and tailored toward financial applications, they have far more repercussions and can be used in wider range of applications in which traditional approaches to risk and uncertainty are under criticism [51].

Conditional value at risk. Conditional value at risk (CVaR) is a coherent risk measure that specifies average loss at a certain confidence level. CVaR has subadditivity property (3.1), a key property lacking in other measures of risk like VaR. α-CVaR by definition is the expected loss under the $(1-\alpha)$-percent of the tail of loss distribution. The attractive mathematical properties and appealing intuitive meaning of CVaR make it an effective risk measure for use in risk-based optimization modeling and decision-making processes. Let $\text{CVaR}_\alpha(X)$ denote the CVaR of X at confidence level of $\alpha \in (0, 1)$, then CVaR can be calculated as follows,

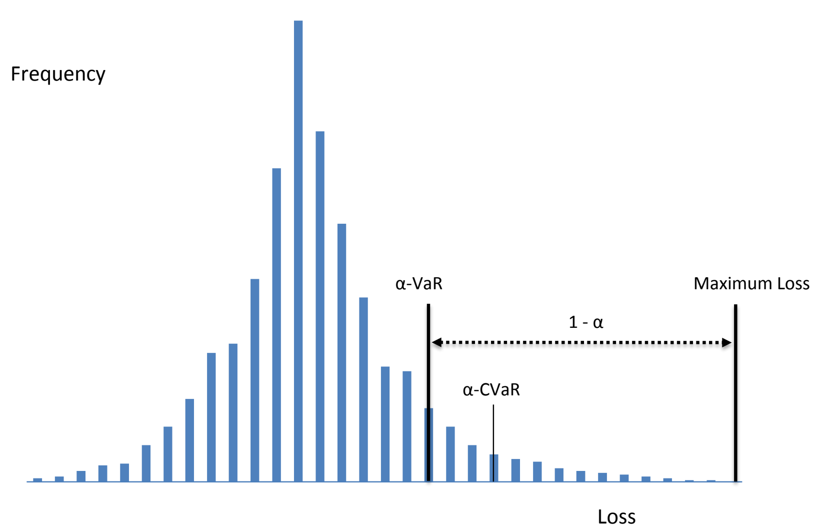


Fig. 3. Value at risk and conditional value at risk.

$$\text{CVaR}_\alpha(X) = \int_{-\infty}^{\infty} y dF_{X,\alpha}(y),$$

where

$$F_{X,\alpha}(y) = \begin{cases} 0, & \text{if} \quad y < \text{VaR}_\alpha(X), \\ (F_X(y) - \alpha)/(1-\alpha), & \text{if} \quad y > \text{VaR}_\alpha(X). \end{cases}$$

Figure 3 depicts the graphical representation of VaR, CVaR and the maximum loss. α-CVaR(X) is continuous with respect to α and can be linearized and used in convex optimization models to restrict the maximum loss to a certain level. CVaR generally performs well when an accurate model or a good estimator of the tail distribution exists [54].

D. Portfolio Risk and Return

Traditionally, portfolios of securities have been constructed using the celebrated mean-variance methodology introduced by Markovitz [40]. In mean-variance portfolio optimization, a portfolio of assets based on the trade-off between the expected returns and risk is selected. The optimization problem consists of finding a portfolio that has the best expected return at a certain level of risk represented by standard deviation of returns. A set of portfolios called the *efficient frontier* can be found by solving this optimization problem for different levels of risk. An efficient frontier consist of portfolios that offer the best mix of risk and return among all levels of risk and return to the investor, *i.e.* provides the highest expected return with a given level of risk. While in finance or any decision-making process that involves monetary values, downside risk is of utmost concern, by using variance to control risk in the mean-variance framework, both upside and downside risks will be affected.

Both CVaR and VaR can be used for limiting downside risks in a portfolio optimization model in the general mean-risk framework in which the trade-off between mean return and a measure of risk are considered. However, CVaR is convex and can be represented with linear constraints in a convex optimization model. Ogryczak and Ruszczynski [45] have demonstrated that a portfolio constructed through the mean-CVaR model satisfies the second order stochastic dominance criterion. Krokhmal et al. [30] have shown that the the mean-risk portfolio problem in which risk is measured by CVaR can be formulated in three different ways and all three will generate the same efficient frontier.

Theorem D.1 *Let us consider the risk function $\phi_\alpha(\boldsymbol{x})$ and reward function $R(\boldsymbol{x})$ dependent on the decision vector $\boldsymbol{x}$, and the following three problems:*

$$(P1) \qquad min_{\boldsymbol{x}} \;\; \phi_\alpha(\boldsymbol{x}) - \mu R(\boldsymbol{x}), \quad \boldsymbol{x} \in \boldsymbol{X}, \;\; \mu \geq 0,$$

$$(P2) \qquad min_{\boldsymbol{x}} \;\; \phi_\alpha(\boldsymbol{x}), \quad R(\boldsymbol{x}) \geq \rho, \quad \boldsymbol{x} \in \boldsymbol{X},$$

$$(P3) \qquad min_{\boldsymbol{x}} \;\; R(\boldsymbol{x}), \quad \phi_\alpha(\boldsymbol{x}) \leq w, \quad \boldsymbol{x} \in \boldsymbol{X}.$$

Varying the parameters μ, ρ, and w, traces the efficient risk-return frontiers for the problems (P1)-(P3), accordingly. If $\phi_\alpha(\boldsymbol{x})$ is convex, R($\boldsymbol{x}$) is concave and the set $\boldsymbol{X}$ is convex, then the three problems, (P1)-(P3), generate the same efficient frontier.

Refer to [30] for the proof.

E. Summary

In this chapter, we presented a formal definition of risk along with standard risk management practices. Further, we described expected utility and certainty equivalent as traditional measures of risk and a coherent risk measure as a modern measure of risk. We provided the definition of coherent risk measures and compared Value at Risk and Conditional Value at Risk as two widely used measure of risk for financial risk management. As a convex measure of risk, CVaR can be incorporated into convex optimization models.

CHAPTER IV

VALUING RISKY PROJECTS IN PARTIALLY COMPLETE MARKETS

This chapter presents a valuation approach for large-scale engineering projects. The value of a project is defined through the difference between the value of an optimal mean-CVaR portfolio of the project and securities and a portfolio that only consists of securities. We show that the value obtained in this way is consistent with the value obtained from standard option pricing. Further, we demonstrate that in this approach, the problem of valuing a project and finding optimal security investment weights can be decomposed into two separate problems and can be solved separately with the output from one being used as input for the other. These two results are presented by Smith and Nau [60] for a decision-maker that uses exponential utility function in maximizing the utility of cash flows. Furthermore, a valuation procedure is introduced that employs CVAR to value the private risk cash flows and market information to value market uncertainties. By following this procedure, a decision-maker can discover the value without constructing an optimal mean-CVaR portfolio of the project and securities. Finally, we present the application of this valuation approach to valuing a transportation PPP.

This chapter is organized as follows. First, background and preliminaries are discussed in Sections A and B. In Section C, mean-CVaR portfolio optimization model is presented. Section D discusses valuation in complete markets and partially complete markets. Section E presents results of a case study, and summary of the results is presented in Section G.

A. Introduction

Capital budgeting or the process of determining the value of a firm's long-term investments, has been the subject of many studies by both decision scientists and financial economists. Discounted Cash Flow (DCF) analysis is the classical approach for economical analysis of investment decisions. In DCF analysis, project's net present value (NPV) is calculated by discounting future cash flows with a risk-adjusted rate of return. The discount rate consists of two elements; risk-free rate and risk premium. Risk-free rate is the rate on a risk-free investment and risk premium is the extra return added to the risk-free rate to justify uncertainty in the cash flows. Finding an appropriate discount rate for a given cash flow stream is a challenging task especially if the asset that generates cash flows is non-tradable. In the context of valuing a risky project, a non-tradable non-diversifiable asset, *e.g.* a large infrastructure project, poses several unique challenges in determining the value of a risky project to a decision-maker. The biggest source of difficulty in valuing such projects comes from the non-diversification and lack of liquidity of private company's assets. Managerial flexibility or real options that give investor the right but not obligation to invest in the project over a certain period of time, are other issues that bring additional complexity to the valuation process.

There are several drawbacks in calculating the value of a risky project using DCF analysis. First, it is usually difficult to find an appropriate discount rate. Two common methods for determining a discount rate are Capital Asset Pricing Model (CAPM) [57] or Weighted Average Cost of Capital (WACC) that is the average rate of company's sources of financing from debt and equity. In CAPM, discount rate is calculated from the risk-free rate and a factor that measures the volatility of underlying asset returns compared to the volatility of publicly traded companies. Basic assumption in CAPM is that the asset will be added to a well-diversified portfolio of assets. This is necessary because the discount rate calculated in this way only accounts for systematic or the overall market risks. CAPM cannot be used for discounting cash flows of a stand-alone risky project since they are susceptible to project-specific uncertainties that cannot be diversified away by constructing a portfolio of publicly traded assets. Finding a discount rate by calculating the weighted average cost of capital may not accurately represent the underlying risks since it is calculated from the firm's historical performance, or from the industry's average WACC. In addition, the NPV calculated in DCF analysis is very sensitive to a small change in the discount rate. Finally, a decision-maker's exposure to risk can not be properly captured in DCF analysis.

Option pricing techniques to value a risky asset based on the Black and Scholes [10] model, in which a project is valued according to a portfolio of securities, is another approach to value risky projects. One of the major assumptions in the option pricing approach is the existence of a twin security or a portfolio of securities that has exactly the same payoffs as the project in every time and state of future. Under this assumption, project risks can be perfectly hedged by trading in securities, and the value of the project is equal to the buying and selling value of the replicating portfolio. Equivalently, a discount rate can be calculated for DCF analysis from the future returns and present value of the replicating portfolio for discounting project cash flows.

One can also use decision analysis to compute the value of a risky project. In decision analysis approach, the value of a project is calculated by using subjective probabilities along with a utility function that measures the utility of decision-maker for consuming cash flows. This approach depends heavily on expected utility axioms originally developed by Neumann and Morgenstern [44]. Aside from difficulties and impracticalities of deriving a utility function and related parameters for a decision-maker, expected utility theory has been under criticism for sometimes being inconsistent in representing decision-maker's risk-aversion, particularly in the case of large wagers [49].

Mattar and Cheah [42] classify different type of risks in an infrastructure project into market, unique and private risks, and suggest that large-scale engineering and infrastructure projects are exposed to high level of private risks. Through an example that involves valuation of a real option for oil exploration, they showed that different risk premiums placed on private risks can significantly change the value calculated in this way. Garvin and Cheah [21] provided a review of conventional valuation techniques for risky assets and applied a valuation approach that relies on a risk-adjusted rate of return on a toll road project in Virginia based on the project's historical data. Zhang [65] presented a financial evaluation method to determine the optimal capital structure and financial viability of infrastructure projects, while Zhao and Tseng [66] discussed the application of stochastic diffusion processes to value flexibility in infrastructure project design and construction.

Smith and Nau [60] showed that when decision analysis or option pricing is properly applied to value a risky project, they both should give the same value if securities markets are complete. Furthermore, they pioneered the definition of market and private uncertainties and presented an integrated valuation procedure in the case of *partially complete markets*, *i.e.*, in which project cash flows can not be fully replicated by a trading strategy. More specifically, markets are partially complete if events in the project provide no information about future market events. In other words, only parts of the project risk that are related to market events can be hedged with trading in market securities.

Smith and McCardle [59] applied this approach to valuing oil contracts. Their analysis takes into account commodity prices, production estimate uncertainties and management's ability to change the production rate and opportunities for price hedging, in valuing oil properties. While this approach is novel in recognizing the effects of market and private risks on project valuation and integrating them into a valuation procedure, it suffers from difficulties and sometimes impracticality of using a utility function, like any other model based on the expected utility. In this chapter, a valuation approach for valuing a risky project in partially complete markets, in which project cash flows cannot be fully replicated, is presented. The goal is to develop a valuation approach that is consistent with the standard option pricing framework and in which decision-maker's risk preference can be expressed in a natural way with a minimum amount of subjectivity.

B. Preliminaries

We study valuation of a risky project from the perspective of a risk-averse owner or a manager whose risk preference reflects those of all stakeholders. Without this assumption it would be difficult, if not impossible, to discover the project's value, since otherwise all risk preferences should be incorporated into the model. Let us assume that state of the world is finite and represented by vector $\boldsymbol{\eta} = (\eta_1, \eta_2, .., \eta_s)$ that can be decomposed into market and private states $\boldsymbol{\eta} = (\boldsymbol{\eta}^m, \boldsymbol{\eta}^p)$, and are revealed at discrete times $t = 0, 1, 2, .., T$. The final state of the world is revealed at time-T; at earlier times, the firm has some information about the final state denoted by time-t state of information ω_t. States of information are subsets of the possible states, a vector of market and private states, *i.e.*, $\omega_t = (\omega_t^m, \omega_t^p)$ that are partitions of $\boldsymbol{\eta}$ and are mutually exclusive and become sequentially smaller with increasing t. Let $x_t(\omega_t)$ be a random variable representing project's cash flow at time t in state ω_t. In addition to the project, the firm can invest in securities by building a portfolio of securities, and rebalance the holdings during the project time period. Trading can take place on $N+1$ securities at discrete time periods with price vector $\mathbf{s}_t(\omega_t) = (s_t^0(\omega_t), s_t^1(\omega_t), ..., s_t^N(\omega_t))$, where $s_t^i(\omega_t)$ is the price of i^{th} security at time t in a given state ω_t and $s_t^0(\omega_t)$ being the risk-free security. We denote by $\boldsymbol{\varphi}_t(\omega_t) = [\varphi_t^0(\omega_t), \varphi_t^1(\omega_t), ..., \varphi_t^N(\omega_t)]$ a trading strategy that is a positive or negative holding of securities from time t to $t+1$.

1. Complete Markets

A complete market is a market in which the price of all goods or services can be discovered. If markets are complete, a complete set of future payoffs for a given risky project can be constructed with existing securities in the market. Denote by $\mathcal{P}$ the set of all payoffs that can be constructed by trading strategy $\boldsymbol{\varphi}_t(\omega_t)$ on $N+1$ securities and M states and $(M \times N+1)$ matrix $\mathbf{S}_t(\omega_t)$ whose rows are $\mathbf{s}_t(\omega_t)$,

$$\mathcal{P} = \{k \in \mathbb{R}^M : k = \mathbf{S}_t(\omega_t)\boldsymbol{\varphi}_t^T(\omega_t),\ \boldsymbol{\varphi} \in \mathbb{R}^{N+1}\}.$$

Security markets are complete if the rank of $\mathbf{S}_t(\omega_t)$ is greater than or equal to M for each t, *i.e.*, portfolio payoffs k can span the space of all possible project cash flows [33]. When markets are complete and there is no opportunity for *arbitrage* (*e.g.* there is no possibility for a positive immediate gain without taking any risk), value of a risky project can be determined from a replicating portfolio based on the *law of one price*. The law of one price states that if there exists a replicating portfolio that, if combined with borrowing or lending, can accurately replicate cash flows of another security, they both should have the same price.

In complete markets, for any given project with no private uncertainty, a portfolio of securities can be constructed that replicates project's cash flows $c_t(\omega_t)$ at all times and all states by the replicating trading strategy $\tilde{\boldsymbol{\varphi}}$ as follows,

$$[\tilde{\boldsymbol{\varphi}}_{t-1}(\omega_{t-1}) - \tilde{\boldsymbol{\varphi}}_t(\omega_t)]\mathbf{s}_t(\omega_t) = c_t(\omega_t), \forall t > 0, \omega_t. \tag{4.1}$$

The final replicating portfolio at time T is a zero-weight holding of securities for all ω_T (the product here is a dot product). In the reminder of this chapter, $\boldsymbol{\varphi}_t(\omega_t)$ and $\mathbf{s}_t(\omega_t)$ are suppressed to $\boldsymbol{\varphi}_t$ and $\mathbf{s}_t$, respectively, for convenience.

2. Risk-neutral Pricing

An alternative and straightforward way of pricing risky assets in complete markets is risk-neutral pricing in which value of the project is determined through a set of risk-neutral probabilities instead of a risk-adjusted rate of return.

Luenberger [36] describes deriving risk-neutral probabilities from replicating portfolio as follows. Define by *elementary state security* a security that has payoff in only one state m, $e_m = (0, \ldots, 0, 1, 0, \ldots, 0)$, where 1 is payoff at state m and has positive price ζ_m. If a complete set of state securities exists (at least one for each state), value of the project c can be obtained from the elementary state securities based on *linear pricing principle*. Linear pricing principle states that if there are two securities with price p_1 and p_2, the price of combined security must be $p_1 + p_2$, otherwise, it will result in an opportunity for arbitrage.

A single-period project c that has payoff $p^m, m \in M$, at state m, can be described as the combination of elementary securities, $c = \sum_{m=1}^{M} p^m e_m$. Therefore, based on the linearity of pricing, price of c must be,

$$v = \sum_{m \in M} p^m \zeta_m. \tag{4.2}$$

Let $\zeta^* = \sum_{m=1}^{M} \zeta_m$, and let $\pi_m = \zeta_m/\zeta^*$. The values π_m that are calculated in this way can be considered as probabilities since they are positive and sum to one. Then, the project value can be obtained by,

$$v = \zeta^* \sum_{m=1}^{M} \pi_m p^m. \tag{4.3}$$

Denote by E_π expectation with respect to π, then we can rewrite Equation (4.3) as,

$$v = \zeta^* E_\pi \left[p\right]. \tag{4.4}$$

Because $\zeta^* = \sum_{m-1}^{M} \zeta_m$, ζ^* can be interpreted as the price of security (1,1,...,1) whose payoff is 1 at every state, *i.e.*, a risk-free security. The price of such security, by definition, is $\frac{1}{R}$ where R is the risk-free return. Therefore, (4.4) can be written as,

$$v = \frac{1}{R} E_\pi \left[p\right]. \tag{4.5}$$

Equation (4.5) states that the value of the project is equal to the discounted expected value of its payoffs under risk-neutral probabilities.

More generally, if there are n states and at least n independent securities for each state with known prices v_i, risk neutral probabilities can be computed by a system of equations as follows,

$$v_i = \frac{1}{R} \sum_{m=1}^{M} \pi_m p_i^m \qquad i = 1, 2, \ldots, n \tag{4.6}$$

for n unknown π_m.

C. Mean-CVaR Portfolio Optimization

As discussed in Chapter III, the objective of mean-CVaR portfolio optimization is to maximize expected return subject to downside risk expressed by CVaR. Denote by $x(t, \boldsymbol{\varphi}_t)$ cash flows generated from $\boldsymbol{\varphi}_t$ at time t,

$$x(t, \boldsymbol{\varphi}_t) = [\boldsymbol{\varphi}_{t-1} - \boldsymbol{\varphi}_t]\mathbf{s}_t,$$

and by $\Psi(\boldsymbol{\varphi}_t, .)$ resulting distribution function of the cash flows at time-t. The α-VaR of the loss associated with the decision variables $\boldsymbol{\varphi}_t$ is given by

$$\zeta_\alpha(\boldsymbol{\varphi}_t) = \min\{\zeta : \Psi(\boldsymbol{\varphi}_t, \zeta) \geq \alpha\}.$$

The α-CVaR $\phi_\alpha(\boldsymbol{\varphi}_t)$ of the loss associated with the decision variable $\boldsymbol{\varphi}_t$ is given by the mean of α-tail distribution of NPV of cash flows. The loss being the negative of returns, $\phi_\alpha(x)$ can be calculated from,

$$\phi_\alpha(\boldsymbol{\varphi}_t) = \min_\zeta F_\alpha(\boldsymbol{\varphi}_t, \zeta),$$

with

$$F_\alpha(\boldsymbol{\varphi}_t, \zeta) = \zeta + \frac{1}{1-\alpha}\mathbf{E}\{[-x(t, \boldsymbol{\varphi}_t) - \zeta]^+\},$$

where $[t]^+ = \max\{0, t\}$. This representation of CVaR is convex and continuously differentiable and can be linearized through auxiliary variables and be used within a linear programming model [52, 53].

We would like to maximize expected NPV of the cash flows generated from the multi-period trading strategy $\boldsymbol{\varphi}$ given the α-CVaR constraint. Denoting by $\Gamma\{x(0, \boldsymbol{\varphi}), x(1, \boldsymbol{\varphi}), ..., x(T, \boldsymbol{\varphi})\}$ the expected NPV of cash flows generated by trading strategy $\boldsymbol{\varphi}$ and α-CVaR risk measure, the mean-CVaR portfolio optimization problem can be formulated as follows,

$$\max \Gamma\{x(0,\boldsymbol{\varphi}), x(1,\boldsymbol{\varphi}), ..., x(T,\boldsymbol{\varphi})\} := \max_{\varphi \in \boldsymbol{\varphi}} \mathbf{E}\left[\sum_{t=0}^{T} \frac{x(t,\boldsymbol{\varphi}_t)}{(1+r_f)^t}\right]$$

$$s.t.$$

$$\sum_{t=0}^{T} \frac{\phi(\boldsymbol{\varphi}_t)}{(1+r_f)^t} \le w,$$

where w is the maximum risk allowed.

1. Breakeven Buying and Selling Prices

Founded on the mean-CVaR portfolio optimization model, we present a general definition for the buying and selling price of a risky project for a risk-averse decision maker. As discussed earlier, this valuation approach is based on the risk attitude of an owner or manager whose risk preference reflects that of all stackholders. Breakeven buying price is the price at which the firm is indifferent between investing in the project and doing nothing. Similarly, breakeven selling price is the price at which the firm is indifferent between selling the project and doing nothing. Let us assume a firm is considering to invest in a risky project that generates risky cash flow stream c_t at time t, while it has the opportunity to invest in security markets. Denote by $x^-(t,\boldsymbol{\varphi}^-)$ the time-t cash flows resulting from investing in securities, and $x^+(t,\boldsymbol{\varphi}^+)$ the time-t cash flows resulting from investing in both the project and securities. Then

$$x^-(t,\boldsymbol{\varphi}^-) = [\boldsymbol{\varphi}^-_{t-1} - \boldsymbol{\varphi}^-_t]\mathbf{s}_t,$$

and

$$x^+(t, \boldsymbol{\varphi}^+) = [\boldsymbol{\varphi}^+_{t-1} - \boldsymbol{\varphi}^+_t]\mathbf{s}_t + c_t.$$

We define the breakeven buying price of the project as the time zero lump-sum value v_b that makes the following equality hold,

$$\begin{aligned}\max_{\boldsymbol{\varphi}^+} \Gamma\{x^+(0, \boldsymbol{\varphi}^+) - v_b, x^+(1, \boldsymbol{\varphi}^+), ..., x^+(T, \boldsymbol{\varphi}^+)\} = \\ \max_{\boldsymbol{\varphi}^-} \Gamma\{x^-(0, \boldsymbol{\varphi}^-), x^-(1, \boldsymbol{\varphi}^-), ..., x^-(T, \boldsymbol{\varphi}^-)\}.\end{aligned} \tag{4.7}$$

Similarly, we define the breakeven selling price of the project as the time zero lump-sum value v_s that makes the following equality hold,

$$\begin{aligned}\max_{\boldsymbol{\varphi}^+} \Gamma\{x^+(0, \boldsymbol{\varphi}^+), x^+(1, \boldsymbol{\varphi}^+), ..., x^+(T, \boldsymbol{\varphi}^+)\} = \\ \max_{\boldsymbol{\varphi}^-} \Gamma\{x^-(0, \boldsymbol{\varphi}^-) + v_s, x^-(1, \boldsymbol{\varphi}^-), ..., x^-(T, \boldsymbol{\varphi}^-)\}.\end{aligned} \tag{4.8}$$

To guarantee that the breakeven buying and selling prices as defined by (4.7) and (4.8) exist, for both definitions, we assume that the problems are feasible *i.e.* the optimal solutions are finite and can be obtained by a trading strategy.

D. Project Valuation

In complete markets, when decision analysis or option pricing is properly applied to value a risky project, the two methods should be consistent in providing the same value. Furthermore, in complete markets, the grand problem of determining value of the project and optimal investment in securities can be divided into two separate problems; 1) investment problem and 2) financing problem. The output from the investment problem, which is the value of the project, can be used for solving the financing problem, which is the optimal portfolio of security holdings [60].

1. Valuation in Complete Markets

In this section, we establish the consistency and separation results for the mean-CVaR approach in complete markets. First, if markets are complete, the buying and selling prices as defined by (4.7) and (4.8) should be equal to the value obtained from standard option pricing approach.

Theorem D.1 (Consistency Theorem) *If security markets are complete, then the firm's breakeven buying and selling prices for any project are both equal to the value given by option pricing.*

Proof Let $x^+(t, \boldsymbol{\varphi}^+)$ be the cash flow from investing in project and securities and $x^-(t, \boldsymbol{\varphi}^-)$ be the cash from from investing only in securities, then we can write $\boldsymbol{\varphi}^+ = \boldsymbol{\varphi}^- - \tilde{\boldsymbol{\varphi}}$. The cash flow from investing in the securities and project is

$$\begin{aligned} x^+(t, \boldsymbol{\varphi}^+) &= [\boldsymbol{\varphi}_t^+ - \boldsymbol{\varphi}_{t-1}^+]\mathbf{s}_t + c_t \\ &= [\boldsymbol{\varphi}_t^- - \boldsymbol{\varphi}_{t-1}^-]\mathbf{s}_t - [\tilde{\boldsymbol{\varphi}}_t - \tilde{\boldsymbol{\varphi}}_{t-1}]\mathbf{s}_t + c_t. \end{aligned}$$

The last two terms cancel each other out by the definition of replicating portfolio according to (4.1) for all $t > 0$. For $t = 0$ we have,

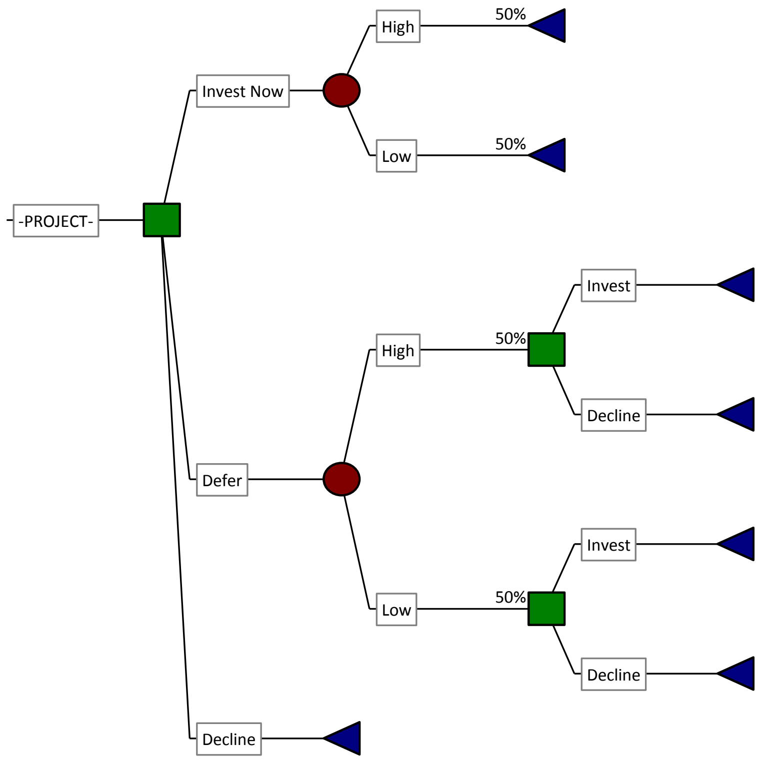
$$x^+(0, \boldsymbol{\varphi}^+) = x^-(0, \boldsymbol{\varphi}^-) + \tilde{\boldsymbol{\varphi}}_0\mathbf{s}_0 + c_0.$$

Therefore, in order for (4.7) to hold, v_b must be equal to $\tilde{\boldsymbol{\varphi}}_0\mathbf{s}_0 + c_0$, which is the time-0 value of replicating portfolio and also the value obtained using standard option pricing. The consistency of breakeven selling price can be proved by the same approach.

To illustrate this result, we use the same capital budgeting example used in the literature. In this example a firm is facing three possibilities and would like to find the value of each possibility. The three possibilities are 1) to invest \$104 in a project that has \$180 payoff in the high state and \$60 in the low state; 2) acquire the option to defer investment for one year. After one year, if market is strong, then the firm can invest in the project or walk away if market is weak; 3) do nothing. Figure 4 depicts the decision tree representing cash flows resulting from each alternative. The numbers in the right show net present values discounted at risk-free rate (8%). We assume that there is a twin security that has the current market price of \$20 and whose payoff is \$36 in the high state and \$12 in the low state. If w_0 and w_1 are the weights of risk-free and twin security in the portfolio, the replicating portfolio for invest now option can be derived from the following system of equations,

$$\begin{cases} w_0(1.08) + w_1(36) = 180 \\ w_0(1.08) + w_1(12) = 60. \end{cases}$$

The solution is $w_0 = 0$ and $w_1 = 5$ and since each share of twin security is priced at \$20, value of the invest option is $5 \times 20 = 100$, minus the time-0 cash flow of \$104 that yields -\$4. Alternatively, risk-neutral probabilities can be derived from $\frac{\pi(36)+(1-\pi)(12)}{1+0.08} = 20$, that will result in $\pi = .4$ and NPV $= \frac{180\times.4+60\times.6}{1.08} - 104 = -4$. According to (4.7), buying price of the project is the value that if added to time-0 cash flows, it sets the difference between the solution of mean-CVaR optimization problems with and without the project to zero. In this case, -4 sets the difference to zero. The project payoffs and CVaR values before valuation are depicted in Figure 5 and the projects payoffs and CVaR values after the valuation is depicted in Figure 6. The expected returns are calculated from the solutions of the optimization problems



Cash Flow Current	Cash Flow Year One	NPV @ 8%
-104	180	62.67
-104	60	-48.44
0	67.7	62.67
0	0	0
0	-52.2	-48.44
0	0	0
0	0	0

Fig. 4. Decision tree.

in (4.7). Table I shows the portfolio weights as the result of investing in only securities; solution of the right optimization problem and Table II shows the portfolio weights by investing in the project and securities; solution to the left optimization problem in (4.7). The difference between NPV of expected returns is zero when the project is valued at -\$4.

			NPV @ 8%	**90%-CVaR**
Project		180	62.667	
	-104			-48.444
		60	-48.444	
Twin Security		36	13.333	
	-20			-8.8889
		12	-8.8889	
Risk-free Security	-1	1.08	1.08	

Fig. 5. Project, twin security and risk-free security cash flows with CVaR values before valuation.

Table I. Portfolio weights by investing in only securities.

	Weights	Expected Return
Twin security	10.125	243.00
Risk-free security	97.50	105.30
Total		348.30

The option pricing value of defer alternative can be obtained from,

$$\begin{cases} w_0(1.08) + w_1(36) = 67.68 \\ w_0(1.08) + w_1(12) = 0, \end{cases}$$

			NPV @ 8%	90%-CVaR
Project		180	66.667	
	-100			-44.444
		60	-44.444	
Twin Security		36	13.333	
	-20			-8.8889
		12	-8.8889	
Risk-free Security	-1	1.08	1.08	

Fig. 6. Project, twin security and risk-free security cash flows with CVaR values after valuation.

Table II. Portfolio weights by investing in the project and securities.

	Weights	Expected Return
Twin security	5.125	123
Risk-free security	97.50	105.30
Project	1	120
Total		348.30

that yields $w_0 = -31.33$ and $w_1 = 2.82$. The value of defer option is the current value of this portfolio, -\$31.33+2.82×(\$20)=\$25.07 that is the same as buying price of the project obtained from (4.7). Figures 7 and 8 demonstrate the project payoffs and CVaR values before and after valuation. Table III shows the portfolio weights as the result of investing in only securities; solution to the right optimization problem in (4.7). Table IV shows the portfolio weights by investing in the defer option and securities; solution to the left optimization problem in (4.7). The difference between the NPV of expected returns is zero when the defer option is valued at \$25.07.

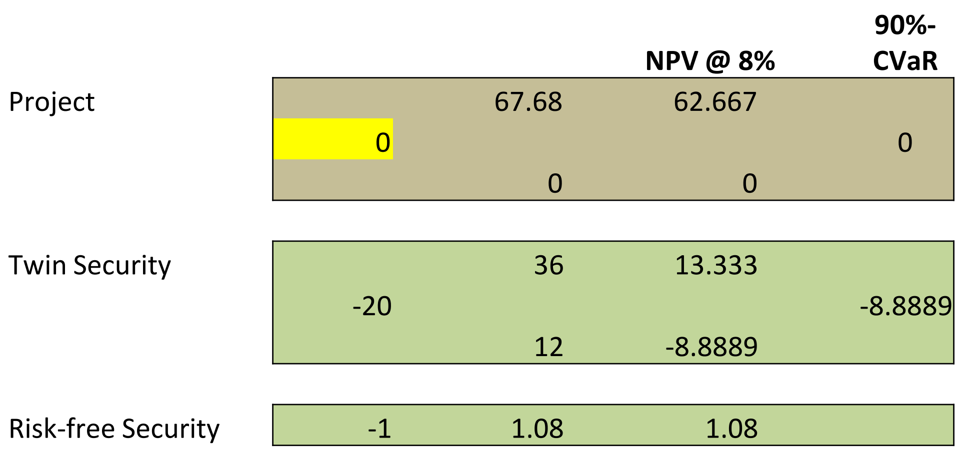


Fig. 7. Defer option, twin security and risk-free security cash flows with CVaR values before valuation.

Table III. Defer option: portfolio weights by investing in only securities.

	Weights	Expected Return
Twin security	6.75	162.00
Risk-free security	165.00	178.20
Total		340.20

			NPV @ 8%	90%-CVaR
Project		67.68	37.6	
	-25.067			-21.933
		0	-25.067	
Twin Security		36	13.333	
	-20			-8.8889
		12	-8.8889	
Risk-free Security	-1	1.08	1.08	

Fig. 8. Defer option, twin security and risk-free security cash flows with CVaR values after valuation.

Table IV. Defer option: portfolio weights by investing in the project and securities.

	Weights	Expected Return
Twin security	3.93	94.32
Risk-free security	196.33	212.04
Project	1	33.84
Total		340.20

The best project management strategy can be found by solving the investment problem that is the alternative with the highest value. Further, the results of the investment problem can be used to solve the financing problem whose output is optimal weight of securities.

Theorem D.2 (Separation Theorem) *Given a risky project c, if the securities market is complete, let κ^* be a maximizing project strategy obtained from*

$$v^* = \max_{\kappa} E_\pi[\sum_{t=0}^{T} \frac{c_t^\kappa}{(1+r_f)^t}],$$

and let $\boldsymbol{\varphi}^{r^*}$ be the replicating trading strategy for c^{κ^*}. Let $\boldsymbol{\varphi}^{f^*}$ denote a trading strategy that maximizes,

$$\max_{\varphi^f} \Gamma\{x_f(0,\boldsymbol{\varphi}^f) + v^*, x_f(1,\boldsymbol{\varphi}^f), ..., x_f(T,\boldsymbol{\varphi}^f)\},$$

where

$$x_f(t,\boldsymbol{\varphi}^f) = [\boldsymbol{\varphi}_{t-1}^f - \boldsymbol{\varphi}_t^f]\boldsymbol{s}_t.$$

Then κ^* and $\boldsymbol{\varphi}^{g^*} = \boldsymbol{\varphi}^{f^*} - \boldsymbol{\varphi}^{r^*}$ solves the grand problem

$$\max_{\varphi^g,\kappa} \Gamma\{x_g(0;\kappa,\boldsymbol{\varphi}^g), x_g(1;\kappa,\boldsymbol{\varphi}^g), ..., x_f(T;\kappa,\boldsymbol{\varphi}^g)\},$$

where

$$x_g(t,\kappa,\boldsymbol{\varphi}^g) = [\boldsymbol{\varphi}_{t-1}^g - \boldsymbol{\varphi}_t^g]\boldsymbol{s}_t + c_t^\kappa.$$

Proof For $\kappa = \kappa^*$ and $\boldsymbol{\varphi}^g = \boldsymbol{\varphi}^f - \boldsymbol{\varphi}^{r^*}$, the time-$t$ cash flows generated from the grand problem is $x_g(t;\kappa^*,\boldsymbol{\varphi}^g) = [\boldsymbol{\varphi}_{t-1}^f - \boldsymbol{\varphi}_t^f]\mathbf{s}_t - [\boldsymbol{\varphi}_{t-1}^{r^*} - \boldsymbol{\varphi}_{t-1}^{r^*}]\mathbf{s}_t + c_t^{\kappa^*}$. The last two terms cancel by the definition of replicating portfolio according to (4.1) for all $t > 0$. For $t = 0$, $x_g(t;\kappa^*,\boldsymbol{\varphi}^g) = x_f(0,\boldsymbol{\varphi}^f) + v^*$, which is the exact cash flows as the financing problem. Therefore, $\boldsymbol{\varphi}^{g^*} = \boldsymbol{\varphi}^{f^*} - \boldsymbol{\varphi}^{r^*}$.

In the capital budgeting example, the optimal project management strategy is to acquire the defer option since the value of invest now is -4 and the value of defer option is 25.07. Given that deferring the investment is the optimal strategy, the financing problem can be solved to find the optimal weight of securities by adding this value to the time-0 cash flows. The optimal weights are $w_0 = 6.75$ and $w_1 = 165$. The replicating portfolio for defer option as obtained earlier is $w_0 = -31.33$ and $w_1 = 2.82$. According to the separation theorem, the optimal weight of securities in the grand problem can be obtained by subtracting the solution of financing problem from replicating portfolio that yields $w_0 = 165 + 31.33 = 196.33$, $w_1 = 6.75 - 2.82 = 3.93$. These weights are exactly the weights that are found and shown in Table IV.

2. Valuation in Incomplete Markets

In general, when markets are incomplete, *i.e.*, when state prices are not unique, one can not find a unique value for a risky project based on the risk-neutral probabilities since these probabilities are no longer unique. These probabilities are not unique since the market prices do not span the space of all possible project cash flows, a condition that was required for complete markets and discussed in Section 4.6. When markets are incomplete, only an upper and lower bound can be established for the project's value. These bounds are obtained from calculating the expected value with respect to a set of probability distributions generated from risk-neutral pricing approach. The bounds that can be obtained for the project with a risky cash flows of c_t and a set of risk-neutral distributions $\pi \in \Pi$ are,

$$\hat{v} = \sup \mathrm{E}_\pi \left[\sum_{t=0}^{T} \frac{c_t}{(1+r_f)^t} \right]$$

and

$$\check{v} = \inf \mathrm{E}_{\pi} \left[\sum_{t=0}^{T} \frac{c_t}{(1+r_f)^t} \right].$$

3. Valuation in Partially Complete Markets

Assuming that the project risk can be decomposed into market and private risks stemming from market uncertainties and firm's private uncertainties, we will extend mean-CVaR valuation presented in the previous section to the case of partially complete markets. As discussed earlier, markets are partially complete if the events in the project provide no information about future market events and only parts of the project risk that are related to market events can be hedged with trading in market securities. To generalize the mean-risk model to partially complete markets, we first define $\phi[.|\omega_t^m, \omega_{t-1}]$.

Definition $\phi_t[.|\omega_t^m, \omega_{t-1}]$ is the time-t CVaR of the private risk cash flows.

Given the definition of time-t CVaR of private risk cash flows, we can define the CVaR replicating strategy $\tilde{\boldsymbol{\varphi}}_t$ in partially complete markets.

Definition $\tilde{\boldsymbol{\varphi}}_t$ replicating strategy in partially complete markets is a replicating strategy that matches the time-t CVaR value in all market states. The final replicating strategy at time T $\tilde{\boldsymbol{\varphi}}_{\omega_T} = 0$. For any earlier time, given $\tilde{\boldsymbol{\varphi}}_t(\omega_t)$ for all ω_t , $\tilde{\boldsymbol{\varphi}}_{t-1}(\omega_t)$ can be obtained from,

$$\tilde{\boldsymbol{\varphi}}_{t-1}\mathbf{s}_t(\omega_t^m) = \phi_t[c_t(\omega_t) + \tilde{\boldsymbol{\varphi}}_t(\omega_t)\mathbf{s}_t(\omega_t^m)|\omega_t^m, \omega_{t-1}]. \tag{4.9}$$

For every ω_{t-1} (4.9) defines a set of K equations for $N+1$ unknowns, where K is the number of market states ω_t^m in ω_{t-1}.

As in complete markets, the project is valued at the market value of the CVaR replicating strategy at time-0 plus any cash flow from the project at time-0. However, to value the project according to breakeven buying and selling prices as defined by (4.7) and (4.8), in partially complete markets, the cash flows generated from investing in both the project and securities should be refined as follows,

$$x^{+}(t, \boldsymbol{\varphi}^{+}) = [\boldsymbol{\varphi}^{+}_{t-1} - \boldsymbol{\varphi}^{+}_{t}]\mathbf{s}_t + y_t(\omega^m_t),$$

where

$$y_t(\omega^m_t) = \phi_t[c_t|\omega^m_t, \omega^p_{t-1}].$$

In this definition, c_t is replaced by $y_t(\omega^m_t)$ because private risk cash flows are measured by CVaR for every market state. This definition generalizes the earlier definition of $x^{+}(t, \boldsymbol{\varphi}^{+})$, as in the absence of private uncertainties, $y_t(\omega^m_t) = c_t(\omega_t)$. The cash flows from investing in securities stays the same as the private risk cash flows are only defined over the project cash flows.

To demonstrate valuation in partially complete markets, private uncertainties are added to the the capital budgeting example discussed before. The updated decision tree is depicted in Figure 9. This figure shows the invest now alternative cash flows for a decision-maker that, in addition to market uncertainties, is faced with private risk stemmed from uncertainty in the firm's efficiency.

Let us assume that the decision-maker's preference for the private risk cash flows is expressed by 40%-CVaR ($1 - \alpha = 40\%$). In other words, the decision-maker considers only average over the 40% of the tail distribution of cash flows. The first equation is written for 40%-CVaR of the private risk cash flows for the high state that is 170, and the second equation is 40%-CVaR for the low state that is 45. In this case, the CVaR replicating strategy can be obtained from the following system of equations,

$$\begin{cases} w_0(1.08) + w_1(36) = 170 \\ w_0(1.08) + w_1(12) = 45. \end{cases}$$

The solution is to buy 5.20833 shares of the twin security and borrow 16.204 shares of risk-free security. The value of the project is the time-0 value of the CVaR replicating portfolio plus project's time-0 cash flow, $(20 \times 5.20833 - 16.204) - 104 = -16.037$ that is the same as the buying price of the project obtained from (4.7).

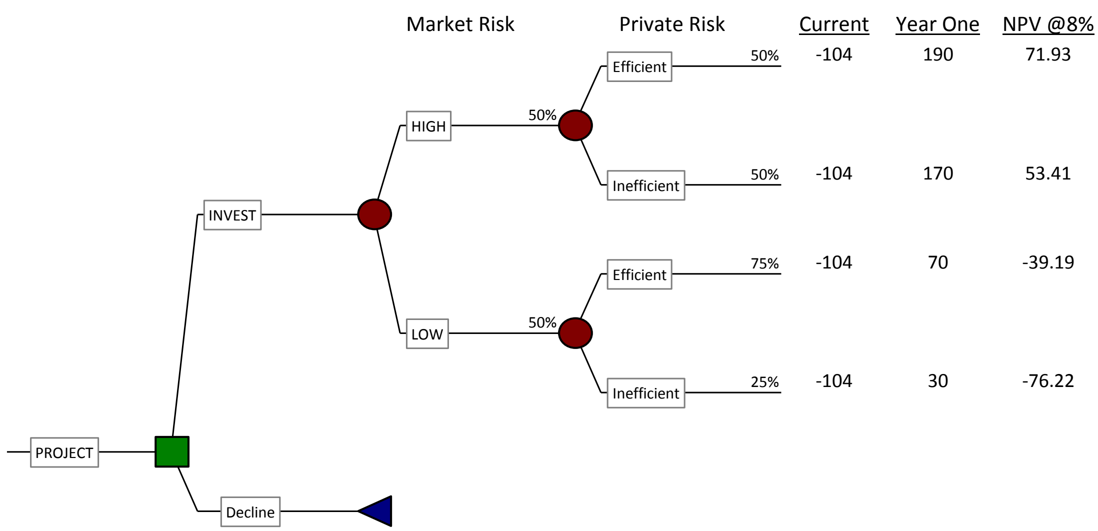


Fig. 9. Decision tree with private uncertainty.

Figures 10 and 11 show the cash flows of project and their corresponding CVaR values before and after valuation, respectively. Table V shows the portfolio weights as the result of investing in only securities; solution to the right optimization problem in (4.7). Table VI shows the portfolio weights by investing in the project and securities; solution to the left optimization problem in (4.7). The difference between the NPV of expected returns is zero when the project is valued at -\$16.037.

					NPV @ 8%	40%-CVaR
			0.5	190	71.9259	53.4074
		0.5	0.5	170	53.4074	
Project	-104					-50.7593
		0.5	0.75	70	-39.1852	
			0.25	30	-76.2222	-62.3333
Twin Security		0.5	36		13.3333	
	-20					-8.8889
		0.5	12		-8.8889	
Risk-free Security	-1		1.08		1.08	

Fig. 10. Cash flows with CVaR values in partially complete markets before valuation.

					NPV @ 8%	40%-CVaR
			0.5	190	87.9630	69.4444
		0.5	0.5	170	69.4444	
Project	-87.963					-34.7222
		0.5	0.75	70	-23.1481	
			0.25	30	-60.1852	-46.2963
Twin Security		0.5	36		13.3333	
	-20					-8.8889
		0.5	12		-8.8889	
Risk-free Security	-1		1.08		1.08	

Fig. 11. Cash flows with CVaR values in partially complete markets after valuation.

Table V. Partially complete markets: portfolio weights by investing in only securities.

	Weights	Expected Return
Security	10.125	243.00
Risk-free security	97.50	105.30
Total		348.30

Table VI. Partially complete markets: portfolio weights by investing in the project and securities.

	Weights	Expected Return
Twin security	4.9167	109.26
Risk-free security	113.7037	122.80
Project	1	107.50
Total		348.30

4. Coherent Valuation Procedure

In this section, we present a valuation procedure called Coherent Valuation Procedure (CVP) for finding the value of a risky project in partially complete markets. To start this procedure, first NPV of all end points in the decision tree is calculated, then the following steps are followed,

1. Upon reaching a node in the decision tree representing a private uncertainty, replace the node with CVaR_t value denoted by $\phi[.|.]$;

2. Upon reaching a node in the decision tree representing a market uncertainty, replace the node with the expected value computed using the risk-neutral probabilities; and

3. Upon reaching a decision node, choose the branch with the maximum value.

Proposition D.3 *The time-t value obtained from the coherent valuation procedure is equal to* $c_t + \tilde{\boldsymbol{\varphi}}_t \boldsymbol{s}_t$.

Proof By induction. Assuming it is true for t, we prove it for $t-1$. The statement is true for $t = T$ since $\tilde{\boldsymbol{\varphi}}_T = \mathbf{0}$. For $t-1$ we have,

$$\begin{aligned} c_{t-1} + \tilde{\boldsymbol{\varphi}}_{t-1} \boldsymbol{s}_{t-1} &= c_{t-1} + \mathbf{E}_\pi \left[\frac{1}{1+r_f} \tilde{\boldsymbol{\varphi}}_{t-1} \mathbf{s}_t | \omega_{t-1} \right] \\ &= c_{t-1} + \mathbf{E}_\pi \left[\phi[\frac{1}{1+r_f} (c_t + \tilde{\boldsymbol{\varphi}}_t \mathbf{s}_t) | \omega_t^m, \omega_{t-1}^p] | \omega_{t-1} \right] \\ &= c_{t-1} + \mathbf{E}_\pi \left[\phi[\frac{1}{1+r_f} v_t | \omega_t^m, \omega_{t-1}^p] | \omega_{t-1} \right] \\ &= v_{t-1}. \end{aligned}$$

The first equality is by completeness of the project with respect to market uncertainties and the existence of unique risk-neutral probabilities. The second equality is by the definition of $\tilde{\boldsymbol{\varphi}}$, and the third equality follows from the induction assumption.

Theorem D.4 (Consistency Theorem) *If the securities market is partially complete and the firm's risk preference is expressed by conditional value at risk, then the firm's breakeven buying price for any project is equal to the value given by the coherent valuation procedure.*

Proof Let us assume there exist another project $\tilde{c}$ that is identical to c except for time-0 cash flow, $\tilde{c}_0 = c_0 - v$. By Proposition D.3, $\tilde{v} = 0$ that means the firm is indifferent between investing in $\tilde{c}$ and a time-0 payment of zero. Therefore, v is the breakeven buying price of c.

To demonstrate the consistency theorem in partially complete markets, we use 40%-CVaR for private risk and risk-neutral probabilities for market risk. In this case, the value obtained for the project from CVP is (170×.4 + 45×.6)/1.08 - 104 = -16.037, which is equal to the buying price of the project shown in Figures 6 and 7 (-104+87.963= -16.037). Figure 12 depicts different values generated from CVP for varying values of confidence level. It also demonstrates the values obtained from integrated valuation procedure (IVP) for a wide range of risk parameter values. As it can be seen in the picture, while IVP yields a narrow range of values for a wide range of risk paraments, CVP shows a better degree of sensitivity with respect to change in α and provides a better picture of project's inherent risk. Furthermore, both procedures yield the same maximum and minimum values: when the decision-maker is risk-neutral, namely the risk parameter in the exponential utility function converges to $+\infty$ in IVP and α=100% in CVP, both procedures yield the same value of -4. In this case, the decision-maker does not require a risk premium for the private risk cash flows. Also, they both give the solution of -24 as utility function parameter converges to $-\infty$ and α to 0%.

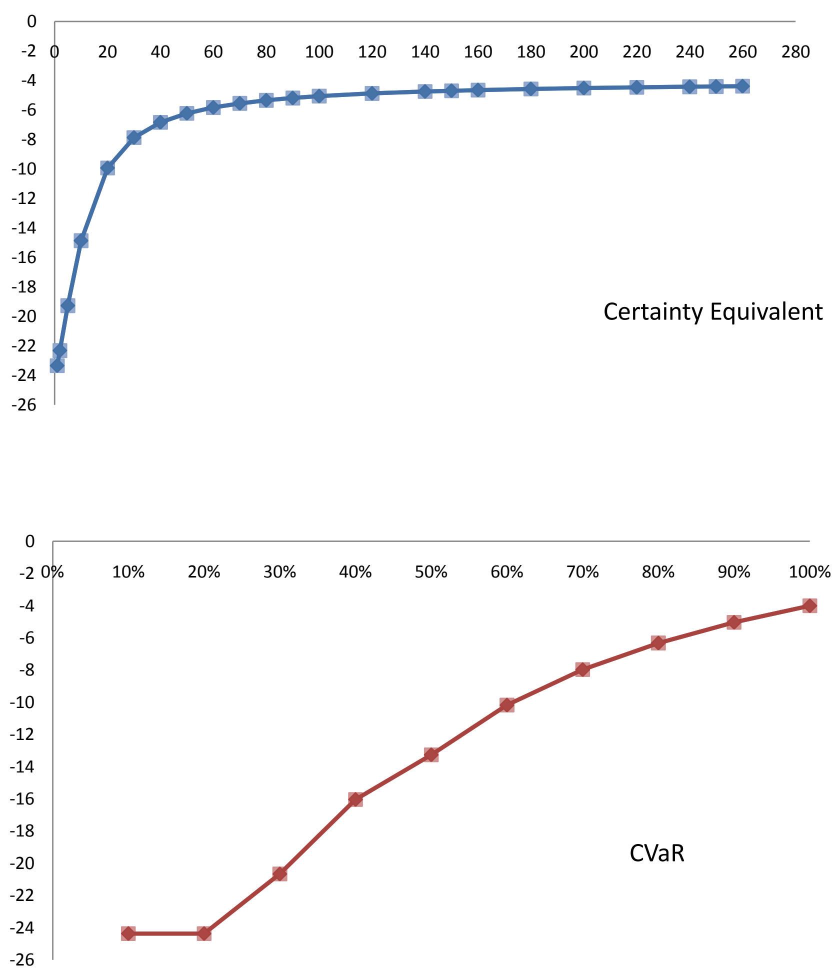


Fig. 12. IVP vs. CVP.

E. Discussion

Capital budgeting or making the decision to invest in a risky project by a manger can expose the firm to a substantial amount of risk. This is particularly important if the investment under consideration is a large engineering projects and involves a significant equity commitment. Traditionally, utility theory and certainty equivalent have been used to measure and capture risk in gambles that consist of private risks. However, values obtained from the certainty equivalent of expected utility, namely an exponential utility function, do not show much sensitivity to the change in the decision maker's risk tolerance represented by utility function parameters. Conditional value at risk is a measure of risk that is used in financial industry to measure downside risks. Having superior properties compared to VaR, like convexity and subadditivity, CVaR can be used in the context of a mean-risk model for valuing risky projects from a perspective of a risk-averse decision-maker without the need for deriving a utility function and related parameters.

If project's uncertainties are well defined and can be decomposed into market and private uncertainties represented by a set of subjective probabilities based on firm's past performance or expert option, one can use CVaR as a surrogate for private risk and standard risk-neutral probabilities for market risk. While risk-neutral probabilities can be used to account for market risk, subjective probabilities along with α percentile are used for valuing private risks. Compared to an exponential utility function, using CVaR for valuing private risk cash flows results in lower degree of subjectivity as the only parameter used in deriving the value is α percentile.

A fundamental concept in finance is proportionality of risks and prospective returns. Without knowing the magnitude of possible losses at different stages of project development, it would be dangerous if not impractical to evaluate or place an appropriate value on a risky asset. The valuation procedure presented in this chapter provides a window of opportunity for the decision-maker to capture and observe the extreme losses by measuring the downside risks at different points of time, making it an effective tool for decision-makers in determining the value of risky projets.

F. Case Study: Dulles Greenway Project in Virginia

In this section, we discuss the application of CVP to valuing a transportation infrastructure project that was considered for public-private partnership in Virginia. State of Virginia is one of the pioneering states in privatizing public infrastructure projects. The Public Private Transportation Act (PPTA) by Virginia Department of Transportation (VDOT) allows VDOT to collaborate with private sector to develop and/or operate transportation projects. One of this projects was a toll highway project that was later named Dulles Greenway by the consortium that developed this project.

1. Project Background

Dulles Greenway project was among the first transportation projects in the U.S. to be developed through Build, Operate and Transfer (BOT) arrangement discussed in Chapter II. Planning for this project started in 1987 after Virginia General assembly authorized the project for PPP due to $7 billion budget deficit. Project development was awarded to a private consortium, allowing them to collect toll on the highway for 40 years. Project background information and estimates discussed in this section are adopted from [21] and are based on financial report submitted by the consortium to the state.

According to the initial plan, construction was planned to start in 1989 and operations in 1992, but due to financing and environmental clearance issues construction did not start until 1993. However, construction finished ahead of schedule in September 1995. The consortium's initial ridership estimates were approximately 20,000 vehicles per day for the first year at the toll rate of $1.5, with projections for ridership increasing to 34,000 vehicle per day at the same toll rate after three years. However, in light of optimism over economic development in the area, ridership estimates revised upward to 34,000 vehicle per day for 1995 and initial toll rate estimate increased to \$2 as project construction started two years after the original date. The total construction costs were approximately \$279 million, of which approximately \$40 million was provided by equity investors, and the rest financed with long-term fixed rate bonds.

Once the highway became operational, the average traffic demand was at a low rate of 10,500 vehicle per day. Consequently, the toll rate was reduced to \$1.00 in March 1996 and projected toll hikes were postponed to spur demand. Ridership increased to 21,000 daily travelers by 1996; however, the effect on projected revenues was not substantial as reduced toll rate offset the increase in ridership. As the result, in summer 1996, project developers started a negotiation process with project creditors for deferring loan payments and debt restructuring.

2. Analysis

The analysis is based on the following assumptions by developers:

1. Average daily traffic demand will grow at 14% annually for the first six and 7% per year for the following 34 years.
2. The toll rates begin at \$2.00 and by the 15th year of operation, rises to \$3.00.
3. Operating expenses start at \$9 million per year and grow at 5% per year.
4. Capital improvements include road maintenance and capacity expansion activities.

The construction and development costs of \$279 million are divided over the two-year construction period, and financing, taxes, depreciation and other costs are ignored for simplification. To calculate project's cash flows, capital expenditures

Table VII. Operating and capital projections.

Period	Average daily traffic	Toll per vehicle	Gross revenue	Operating expenses	Capital improvement	EBIT	Capital expenditure
1							-$139,500,000.00
2							-$139,500,000.00
3	$20,000.00	$1.00	$7,300,000.00	$9,000,000.00		-$1,700,000.00	
4	$22,800.00	$1.00	$8,322,000.00	$9,450,000.00		-$1,128,000.00	
5	$25,992.00	$1.00	$9,487,080.00	$9,922,500.00		-$435,420.00	
6	$29,630.88	$1.25	$13,519,089.00	$10,418,625.00		$3,100,464.00	
7	$33,779.20	$1.25	$15,411,761.46	$10,939,556.25		$4,472,205.21	
8	$38,508.29	$1.50	$21,083,289.68	$11,486,534.06		$9,596,755.61	
9	$41,203.87	$1.50	$22,559,119.95	$12,060,860.77		$10,498,259.19	
10	$44,088.14	$1.50	$24,138,258.35	$12,663,903.80	-$3,000,000.00	$8,474,354.55	
11	$47,174.31	$1.75	$30,132,592.51	$13,297,098.99		$16,835,493.51	
12	$50,476.52	$1.75	$32,241,873.98	$13,961,953.94	-$1,700,000.00	$16,579,920.04	
13	$54,009.87	$1.75	$34,498,805.16	$14,660,051.64		$19,838,753.52	
14	$57,790.56	$2.00	$42,187,110.31	$15,393,054.22		$26,794,056.09	
15	$61,835.90	$2.00	$45,140,208.04	$16,162,706.93		$28,977,501.10	
16	$66,164.41	$2.00	$48,300,022.60	$16,970,842.28	-$9,400,000.00	$21,929,180.32	
17	$70,795.92	$2.25	$58,141,152.20	$17,819,384.39		$40,321,767.81	
18	$75,751.64	$2.25	$62,211,032.86	$18,710,353.61		$43,500,679.24	
19	$81,054.25	$2.25	$66,565,805.16	$19,645,871.30		$46,919,933.86	
20	$86,728.05	$2.50	$79,139,346.13	$20,628,164.86	-$10,400,000.00	$48,111,181.27	
21	$92,799.01	$2.50	$84,679,100.36	$21,659,573.10		$63,019,527.26	
22	$99,294.95	$2.50	$90,606,637.39	$22,742,551.76		$67,864,085.63	
23	$106,245.59	$2.65	$102,766,048.12	$23,879,679.35		$78,886,368.78	
24	$113,682.78	$2.65	$109,959,671.49	$25,073,663.31		$84,886,008.18	
25	$121,640.58	$2.65	$117,656,848.50	$26,327,346.48		$91,329,502.02	
26	$130,155.42	$2.85	$135,394,173.39	$27,643,713.80		$107,750,459.59	
27	$139,266.30	$2.85	$144,871,765.53	$29,025,899.49		$115,845,866.04	
28	$149,014.94	$2.85	$155,012,789.12	$30,477,194.47		$124,535,594.65	
29	$159,445.98	$3.00	$174,593,351.95	$32,001,054.19		$142,592,297.76	
30	$170,607.20	$3.00	$186,814,886.59	$33,601,106.90		$153,213,779.69	
31	$182,549.71	$3.00	$199,891,928.65	$35,281,162.25		$164,610,766.40	
32	$195,328.19	$3.00	$213,884,363.66	$37,045,220.36		$176,839,143.30	
33	$209,001.16	$3.00	$228,856,269.11	$38,897,481.38		$189,958,787.73	
34	$223,631.24	$3.00	$244,876,207.95	$40,842,355.45		$204,033,852.50	
35	$239,285.43	$3.00	$262,017,542.51	$42,884,473.22		$219,133,069.29	
36	$256,035.41	$3.00	$280,358,770.48	$45,028,696.88		$235,330,073.60	
37	$273,957.89	$3.00	$299,983,884.41	$47,280,131.72		$252,703,752.69	
38	$293,134.94	$3.00	$320,982,756.32	$49,644,138.31		$271,338,618.02	
39	$313,654.38	$3.00	$343,451,549.27	$52,126,345.22		$291,325,204.04	
40	$335,610.19	$3.00	$367,493,157.71	$54,732,662.48		$312,760,495.23	
41	$359,102.90	$3.00	$393,217,678.75	$57,469,295.61		$335,748,383.15	
42	$384,240.11	$3.00	$420,742,916.27	$60,342,760.39		$360,400,155.88	
	Present values (@ 8%):		$701,670,806.70	$202,782,824.75	-$19,341,587.31	$490,676,822.68	-$248,765,432.10

including operating expenses and capital improvements are subtracted from annual earnings before interest and taxes (EBIT) over the 40-year concession period as follows,

$$\text{CF} = \text{Gross Revenue - Operating Expenses - Capital Improvements},$$

$$\text{Gross Revenue} = \text{Average Daily Traffic} \times \text{Average Toll Rate} \times 365 \text{ days}.$$

Operating and capital projections are summarized in Table VII.

3. Valuation

To apply CVP for valuing this project based on the projected data, we must first identify different types of risks and categorize them into market and private elements. In this project, the uncertainty in the initial demand is considered a market risk as the initial traffic is directly tied to property development in the area. Previous studies [41] also support the strong relationship between GDP growth and traffic demand in highways. The variation in construction and operating costs are considered as private uncertainty since it is directly related to the developer's ability to execute the project effectively.

Table VIII. Market probabilities.

	Return (%)	Probability
High	27.509	0.363636
Average	11.522	0.363636
Low	-9.716	0.272727

Risk-neutral probabilities for market uncertainties are derived from the S&P 500 index returns between 1960-1992 and are depicted in Figure 13. Table VIII shows

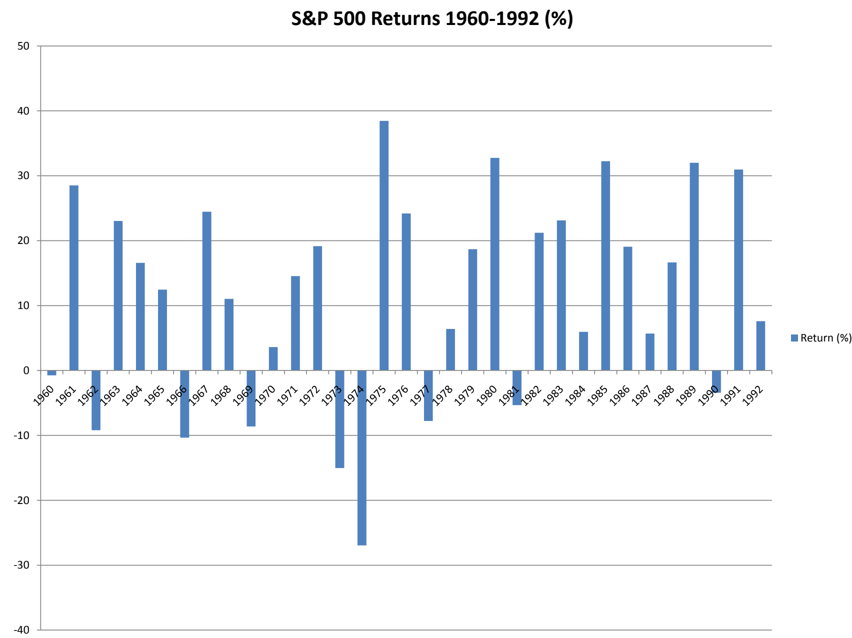


Fig. 13. S&P 500 returns 1960-1992.

the risk-neutral probabilities for the low, average and high market states.

The summary of analysis is tabulated in Table IX. It shows values for three different initial demands of 10,000, 20,000, 30,000 vehicle per day that correspond to low, average and high market states, respectively, and the value obtained under 50%-CVaR. It is assumed that toll price starts at $2 per vehicle if the traffic is high, and gradually increases to $3 in later years. However, if the demand is below 30,000, due to price elasticity it is assumed that toll price starts at $1 and is increased to $3 in a longer time frame.

4. Managerial Implications

Despite the positive NPV obtained from the CVP that are between $278,304,176.49 and $338,436,698.87 and are shown in Table X, analysis reveals that the 90%-CVaR of the project cash flows varies between -$125,884,467.80 to -$186,016,990.18 depending

Table IX. Valuation.

Initial Traffic	Construction and maintenance costs	NPV @8%	50%-CVaR
10000	+30%	-186,016,990.18	
	1	-108,924,012.77	
	-10%	-83,226,353.63	-$159,805,377.86
20000	+30%	164,818,413.17	
	1	241,911,390.58	
	-10%	267,609,049.71	$191,030,025.49
30000	+30%	740,030,814.83	
	1	817,123,792.24	
	-10%	842,821,451.37	$766,242,427.15
		Value	$304,515,788.81

on the choice of α values in CVP. A critical factor in determining the viability of a risky project is the project's exposure to risk. The exposure to risk can be instrumental in the investment decision-making by revealing the amount of risk that the firm is exposed to as the result of investing in this project.

Table X. Value at different α levels.

	Market State			
$1-\alpha$	Low	Average	High	Value
0.1	-$186,016,990.18	$164,818,413.17	$740,030,814.83	$278,304,176.49
0.2	-$186,016,990.18	$164,818,413.17	$740,030,814.83	$278,304,176.49
0.3	-$186,016,990.18	$164,818,413.17	$740,030,814.83	$278,304,176.49
0.4	-$172,525,719.14	$178,309,684.21	$753,522,085.87	$291,795,447.54
0.5	-$159,805,377.86	$191,030,025.49	$766,242,427.15	$304,515,788.81
0.6	-$151,325,150.35	$199,510,253.00	$774,722,654.66	$312,996,016.33
0.7	-$144,166,516.73	$206,668,886.62	$781,881,288.28	$320,154,649.94
0.8	-$136,548,996.34	$214,286,407.00	$789,498,808.66	$327,772,170.33
0.9	-$130,624,258.27	$220,211,145.08	$795,423,546.74	$333,696,908.41
1	-$125,884,467.80	$224,950,935.55	$800,163,337.21	$338,436,698.87

Exposure to risk cannot be observed through traditional methods like valuation through a risk-adjusted discount rate. In fact, the main reason behind financial distress in Dulles Greenway project was due to the inability of the project developers to absorb the losses incurred in the initial years of the project during the time needed for the traffic buildup. In discounted cash flow analysis, not only exposure to risk is not considered in the valuation process, but also the value obtained is very sensitive to the choice of the discount rate. Small change in the discount rate can spell significant change in the NPV of the project. This is even more evident for projects that need some time to become net cash flow positive. For example, assume a project that generates no cash flows for 10 years, and then generates \$1,000,0000 in the 11th year is valued and the discount rate of 30% is used. Discounting cash flows at rate of 30% yields a present value of \$55,798. Now, if the discount rate increases to 32%, the present value falls to \$47,172. This is more than 15% decline in the value as the result of 2% increase in the discount rate.

G. Summary

This chapter introduces a valuation approach based on mean-CVaR portfolio optimization in which a risk-averse decision maker seeks to maximize the expected return subject to downside risk represented by CVaR. We showed that, in complete markets, the value obtained from this approach is equal to the value obtained from standard option pricing approach. Furthermore, if project's uncertainties are well defined and can be decomposed into market uncertainties and private uncertainties represented by a set of subjective probabilities based on firm's past performance or expert option, project's value can be determined by Coherent Valuation Procedure in which CVaR is used as a surrogate for private risk and risk-neutral probabilities for market risk. We showed that the breakeven buying price of a risky project is equal to the value obtained from this valuation procedure.

Comparing the values obtained from this approach and Integrated Valuation Procedure that is based on exponential utility function shows greater sensitivity to the risk tolerance of the decision-maker represented by the parameter α in α-CVaR. In addition, CVP captures the decision-maker's exposure to risk in a more natural way and without using a utility function. As the risk parameter in the exponential utility function converges to infinity reflecting the risk neutrality of the decision-maker, the value obtained from the IVP approach converges to the value of CVP at α=100% that represents risk neutrality in CVP. Finally, the application of CVP is demonstrated through valuing a transportation infrastructure PPP.

CHAPTER V

RISK-BASED MAINTENANCE AND REHABILITATION POLICIES*

A. Introduction

Infrastructure facilities must undergo a number of maintenance and rehabilitation (M&R) actions throughout their life to maximize their service life. Effective maintenance and rehabilitation planning will significantly lower the total life cycle cost of an infrastructure facility, and will also provide a consistent level of service for network users. Many transportation agencies across the nation have developed systems to manage transportation infrastructure assets in a cost-effective and efficient manner. Over the years, a number of models have been developed to determine optimal M&R policies. In such models, facility's deterioration is either considered stochastic with the uncertainty incorporated in the deterioration process [11], or assumed deterministic with future states assumed to be known [46]. Given the impact of uncertainty on the deterioration process and consequently on the M&R optimal policies, the objective of this chapter is to develop a modeling framework for determining long- and short- term policies that take into account the risk associated with deterioration uncertainty.

This chapter is organized as follows. Section B reviews the related literature and introduces the required background,. Section C describes the general approach to risk-based M&R decision making proposed in this paper. It discusses both long- and short-term models developed. To illustrate the approach, we consider a specific

*Parts of this chapter are reprinted with permission from Seyedshohadaie, SR., Damnjanovic, I., Butenko, S.:Risk-based maintenance and rehabilitation decisions for transportation infrastructure networks, Transportation Research, Part A. **44**(4), 236-248 (2010) © Elsevier.

case when risk is defined by pavement deterioration in Section D. In particular, we provide an analytical expression for the risk function and formulate the corresponding LP models. These models are analyzed computationally using a sample network of 20 facilities in Section E. Finally, summary and concludes is discussed in Section F.

B. Literature Review

Markov Decision Process (MDP) is the primary modeling framework for determining the network M&R policies under deterioration uncertainty resulting from different M&R actions. State-of-the-art infrastructure management systems involve MDP models and use dynamic programming techniques to solve for optimal policies. The application of MDP for pavement maintenance was introduced by Arizona Department of Transportation (ADOT) in 1979. Namely, ADOT developed a pavement management system to improve the allocation of its limited resources. The model is based on the data from the inspection of the current condition of facilities, as well as the expectation of the future condition, given an action among a finite set of actions performed on the facility. A detailed description of the model is presented in [22].

M&R policy optimization approaches based on Markov decision process (MDP) framework can be found in [11, 19, 23]. In such approaches, facility conditions are modeled as discrete states and the deterioration process is represented by discrete-time transitional probabilities. The Markovian assumption implies that the condition of the facility at time $t + 1$ depends only on the condition at time t and the action applied to the facility at time t. The optimization model seeks to find minimum cost M&R policy that is a mapping from a set of actions to the set of possible states. Madant and Ben-Akiva [38] and Madanat [37] introduced the latent Markov decision process (LMDP) to account for uncertainly in the current measurement of the facility. Mbwana and Turnquist [43] presented a network-level optimization model based on MDP for pavement management systems with suggestions for the short-term allocation of resources. Guignier and Madanat [25] presented a model for joint optimization of the M&R and improvement policies in a network of infrastructure facilities. Smilowitz and Madanat [58] extended the LMDP model to policies that include network level constraints. Guillaumot et al. [26] combined this model with inspection decisions and presented an adaptive optimization approach to find optimal policies under model uncertainty. A common way of solving such models is by transforming the infinite-time horizon MDP into a linear programming (LP) model, for which efficient algorithms exist. A problem arises when this approach is implemented on the network level in finite-time horizon and under short-term budget restriction. Dynamic programming (DP) is the common approach to optimize MDP in the finite horizon. Adding the budget constraint will result in exponential increase in the number of state variables, making it computationally expensive to solve the model to optimality. Kuhn [31] introduced approximate dynamic programming approach to obtain close to optimal solution for this problem. Therefore, efficient models are needed to address the resource allocation problem.

Another approach to modeling M&R policies is the optimal control framework with continuous maintenance actions and states. The objective of such models is to minimize the total life-cycle costs over a specific time horizon. Tsunokawa and Schofer [62] introduced an optimal control approach to approximate the optimal timing and intensity of maintenance actions. Li and Madanat [35] presented optimal polices under steady-state condition, and Ouyang and Madant [46] provided the exact and approximate rehabilitation intensity and frequency solution approaches assuming finite horizon and deterministic deterioration. Jido et al. [29] used optimal impulse control to solve for optimal repair and inspection policies.

In other proposed methodologies for M&R planning, Durango-Cohen [16] treated deterioration in a time series framework to integrate performance prediction and M&R decision making, Chootinana et al. [12] used a simulation-based genetic algorithm to solve pavement maintenance problem under stochastic deterioration , Ying et al. [64] assumed deterministic model parameters and derived required amount of investment to maintain or improve levels of service for a highway network based on the user equilibrium.

1. Deterioration Uncertainty

Deterioration of transportation infrastructure is a complex process. For example, environmental factors and traffic load are two key factors in the pavement roughness deterioration and are associated with high level of uncertainty and high variance among observations. In MDP-based models, the deterioration process is represented by transitional probabilities. Two common approaches to estimate transitional probabilities are by expert opinion or from empirical data. Both approaches can result in transitional probabilities with high variance in the distributions of future states. Additionally, not all facilities within a given network may have the same transitional probabilities under the same M&R action. In most cases, transitional probabilities should be updated and checked through inspection when the true state of the facilities are revealed.

Several works in the literature specifically address the uncertainty in the pavement performance model. Li et al. [34] discussed the importance of accurate prediction of pavement deterioration in the determination of pavement M&R intensity and frequency policies and presented a nonhomogeneous MDP to determine pavement deterioration rates in different stages. Harper and Majidzadeh [28] used Bayesian strategies to update the parameters of the deterioration model. Kuhn and Mdanat

[32] proposed robust optimization models to obtain policies that are valid for a collection of transitional probabilities. Damnjanovic and Zhang [14] presented a reliability model to predict the pavement performance under uncertain conditions. Durango-Cohen and Madanat [17] introduced an optimal control model, where the uncertainty in the deterioration model is represented by a probability mass function of deterioration rates. In this model, the probability mass function of deterioration rate is updated to increase deterioration rate accuracy. Madanat et al. [39] proposed an open-loop feedback control model, in which model parameters are updated sequentially after every inspection round. Durango-Cohen and Madanat [18] assumed deterioration as an unknown mixture of known performance models taken from a finite set of performance models and presented an optimization model to find joint inspection and maintenance polices. Chu and Durango-Cohen [13] used time series analysis techniques to determine pavement performance model. Deterioration is commonly modeled as an exponential function of time. For pavement roughness, the typical deterministic roughness deterioration model is the following model that is discussed in [35] and [46]:

$$s = s^0 \exp(\xi\tau). \tag{5.1}$$

In this function, ξ is the deterioration parameter; s^0 and s are the roughness at the beginning and the end of the planning horizon, respectively, and $\tau := t - t_0, t > t_0$ is the length of the planning horizon $[t_0, t]$. Assuming that the deterioration parameter ξ is random, the deterioration model in (5.1) provides a more realistic description of the deterioration process. Considering a normally distributed $\xi \sim N(\mu, \sigma^2)$, the random variable $\eta = s/s^0$ has a lognormal probability density function:

$$p_\eta(y) = \frac{1}{y\tau\sigma\sqrt{2\pi}} \exp\left(-\frac{(\ln(y) - \tau\mu)^2}{2(\tau\sigma)^2}\right). \tag{5.2}$$

We would like to utilize a quantitative measure of risk to manage the uncertainty in the deterioration process. This quantitative measure is essential in both finding the optimal steady-state polices that minimize the long-term cost of the network M&R actions and in the short-term budget allocation. The purpose of the next subsection is to introduce and define such measure of risk.

2. Risk-based Resource Allocation

Consider a network of M facilities with $m = 1, .., F$ being the index set of facilities, and define x_m to be the decision variable representing the amount of resources allocated to facility m. Denote by $f_m(x_m, \xi_m)$ the loss function associated with decision variable x_m and the random variable ξ_m representing the uncertain condition state of the facility k after the resource allocation action. The probability density function of ξ_m is assumed to be $p_{\xi_m}(\cdot)$. For a fixed x_m, the probability that $f_m(x_m, \xi_m)$ does not exceed threshold ζ is given by [52]:

$$\Psi_m(x_m, \zeta) = \int\limits_{f_m(x_m, y) \leq \zeta} p_{\xi_m}(y) dy. \tag{5.3}$$

As a function of ζ, $\Psi_m(x_m, \zeta)$ is the cumulative distribution function for the loss associated with x_m and is nondecreasing and assumed to be everywhere continuous on ζ. The α-VaR and α-CVaR for facility k and the loss random variable associated with x_m and any probability level $\alpha \in (0, 1)$ are denoted by $\zeta_\alpha(x_m)$ and $\phi_\alpha(x_m)$, respectively, and are given by

$$\zeta_\alpha(x_m) = \min\{\zeta \in R : \Psi_m(x_m, \zeta) \geq \alpha\}, \tag{5.4}$$

$$\phi_\alpha(x_m) = (1-\alpha)^{-1} \int\limits_{f_m(x_m,y)\geq\zeta_\alpha(x_m)} f_m(x_m,y)p_{\xi_m}(y)dy. \tag{5.5}$$

In order to use (5.5) to calculate α-CVaR for each facility, the probability distribution of states at the end of the planning horizon must be determined. In some cases, when the probability distribution of deterioration function is known, closed form solutions to calculate the α-CVaR in the planning horizon can be derived. In particular, as will be shown in Section D, this is the case for lognormal distribution used in our model to describe the roughness deterioration process. On the other hand, if the probability distribution of deterioration function is not known or such that its α-CVaR cannot be computed analytically, it can be approximated using Monte Carlo simulation. The reader is referred to [53] for a detailed discussion on mathematical foundations behind the concept of CVaR for general loss distributions.

C. Risk-based M&R Decisions

By using the quantitative risk measure described in the previous section, optimal M&R policies that will satisfy a certain level of risk can be determined in both long- and short- term bases. Long-term decisions are needed for the budget planning process, in which the total long-term budget is determined for facilities with initial conditions satisfying the risk level whereas short-term decisions are needed to allocate resources under budget constraints. Figure 14 describes the overall approach to determining risk-based M&R actions for both long- and short- term decisions using CVaR as the measure of risk. First, the distribution of future states in the planning horizon under each M&R action is determined for facilities with conditions below the risk threshold. Then, from these distributions, the actions that satisfy the CVaR level with their corresponding transitional probabilities are derived. Based on this data, the long-term model produces the steady-state risk-averse solution assuming no budget constraints. This solution is the optimal risk-averse action for keeping the facility at the preset risk level within the planning horizon. The state-action mapping obtained from the long-term model is only applied to facilities that are not in failed condition i.e. they are not deteriorated beyond the risk level. At any given year, the transportation agency reviews the facilities that are due for M&R action and if there is a budget shortage for implementing optimal actions, a resource allocation decision is made by solving one of the short-term models. Short-term models are deployed to allocate resources among all facilities that are set for M&R action including the ones that are deteriorated beyond the risk level. Each model will be discussed in greater detail in the remainder of this section.

1. Long-term Risk-averse Decisions

Transportation agencies are interested in policies that maintain the network quality at an acceptable level and are in accordance with their long-term budget projections. Developing long-term M&R policies is the initial step towards this objective.

The proposed risk-based long-term M&R optimization model is formulated in the MDP framework. A Markov Decision Process can be described by a tuple $(\mathcal{S}, \mathcal{A}, \mathcal{P}_a(\cdot,\cdot), \mathcal{C}_a(\cdot))$, where $\mathcal{S} = \{1, \ldots, K\}$ is the finite state space, $\mathcal{A} = \{1, \ldots, N\}$ is the finite action space. $\mathcal{P}_a(i, j)$ is the probability that action a in state i at time t will lead to state j at time $t + 1$. $\mathcal{C}_a(i)$ is the cost obtained as a result of applying action a in state i. The objective of most MDP-based models in the literature is to minimize the total cost that consists of both agency and user costs. While user cost can be included in the model, we are specifically seeking the policies that constrain certain risk in the performance indicator. In this model, instead of seeking a compromise between user cost and agency cost, optimal M&R decisions are generated

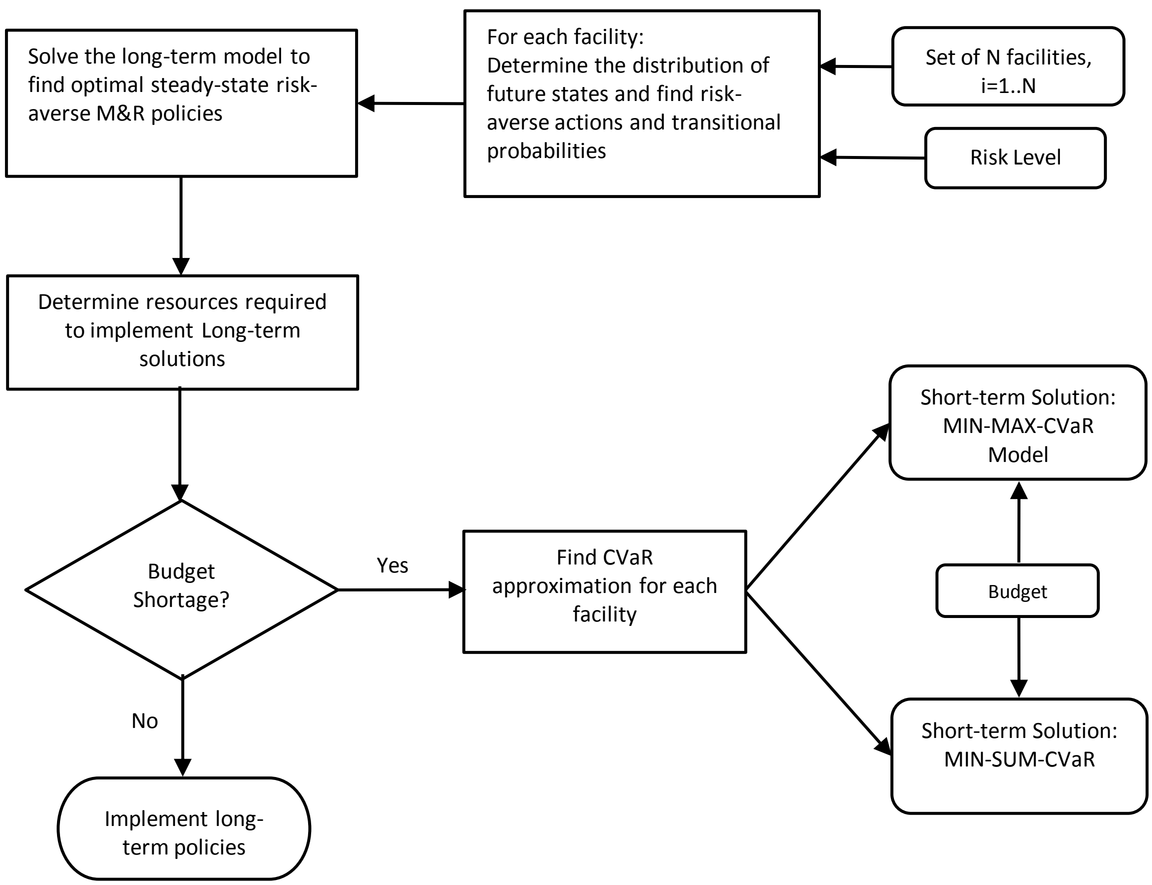


Fig. 14. Overall decision making model.

based on the predefined performance level under risk to avoid the subjectivity that user cost brings to the model.

To develop the optimization model, two main inputs in the MDP must be modified. First, the action set must only consist of actions that always satisfy the prespecified level of risk. Second, the transitional probabilities corresponding to the actions in the action set should be computed by discretizing the probability distribution of the future condition resulting from applying a given action.

2. Risk-averse Actions

To construct a feasible risk-averse action set, the action set is first filtered against the targeted risk level starting from the worst initial state to ensure the targeted risk level is achievable. Then for each initial state and action, actions that satisfy the risk level are determined by the following steps:

1. Derive (or simulate) the distribution of future states;
2. Calculate the $\alpha - CVaR$ for the corresponding risk function distribution; and
3. Check the $\alpha - CVaR$ against the targeted risk level.

Depending on the initial condition, each action will have a different cost due to different M&R intensity required to satisfy the risk constraints. The new action set $\mathcal{A}'$ includes actions that range from the minimum M&R intensity that will satisfy the risk requirement, to the maximum feasible M&R intensity. Finally, from the distribution of facility conditions with an initial state under application of M&R actions, for each initial state the corresponding transitional probabilities and cost of action are computed.

The optimal risk-averse policies can be obtained by solving the MDP with the new transitional probabilities for risk-averse actions. If the distribution of facility condition under different M&R actions is continuous, the state-space must be discretized by a finite grid approximation to solve the model under the MDP framework.

In the next section, a linear programming formulation of MDP for the long-term steady-state risk-averse policies is presented.

3. Long-term Model Formulation

Let $S_t, t = 0, 1, \ldots,$ denote the random variables representing initial state of the facility at time t. We assume the following Markov property:

$$\mathcal{P}_a(i,j) = \mathcal{P}(S_{t+1} = j | S_t = i,\ A_t = a) = \mathcal{P}(S_{t+1} = j | S_t = i,\ A_t = a, S_{t-1}).$$

Let $v_i^n(\pi_\alpha)$ be the value function corresponding to the discounted expected cost of the facility with initial state $i \in \mathcal{S}$, over n periods under the risk-averse policy π_α where α represents the confidence level used in α-CVaR. If the facility is in state i then $a_i(\pi_\alpha)$ is the risk-averse action applied in state i according to the risk-averse policy π_α. To simplify the notation, we will write π_α instead of $a_i(\pi_\alpha)$ whenever it is clear that the policy π_α is applied at state i. We will also write $\pi_\alpha \in \mathcal{A}'$ instead of $a_i(\pi_\alpha) \in \mathcal{A}'$ in such cases. Let $\mathcal{C}_i(\pi_\alpha)$ be the cost of risk-averse M&R policy π_α in state i and let $0 \leq \lambda < 1$ be the discount factor. Then the value function is given by

$$v_i^n(\pi_\alpha) = \mathcal{C}_i(\pi_\alpha) + \lambda \sum_{j \in \mathcal{S}} v_j^{n-1}(\pi_\alpha) \mathcal{P}_{\pi_\alpha}(i,j) \tag{5.6}$$

We assume that the value function is bounded and non-increasing, $\lim\limits_{n \to \infty} v_i^n(\pi) = v_i(\pi)$ exist and under any stationary policy the resulting Markov chain is irreducible. In a stationary infinite-horizon MDP, the steady-state probabilities are independent of the initial states and the steady-state value function for a policy π_α is defined by [48]

$$v_i(\pi_\alpha) = \mathcal{C}_i(\pi_\alpha) + \lambda \sum_{j\in\mathcal{S}} v_j(\pi_\alpha)\mathcal{P}_{\pi_\alpha}(i,j), \tag{5.7}$$

and the long-term optimal solution π_α^* can be derived from

$$a_i(\pi_\alpha^*) = \arg\min_{\pi_\alpha} \left[\mathcal{C}_i(\pi_\alpha) + \lambda \sum_{j\in\mathcal{S}} v_j(\pi_\alpha)\mathcal{P}_{\pi_\alpha}(i,j)\right]. \tag{5.8}$$

Let $v_i^* = v_i(\pi_\alpha^*)$ denote the optimal value of the value function for state i.

One of the standard methods used for solving reasonably large instances of discounted MDP (5.8) is by conversion to linear programming. The derivation of the corresponding linear programming formulation of the discounted MDP is described in detail in [48] and is only briefly sketched here. Denote by $\gamma_j,\ j \in \mathcal{S}$ the probability of starting from initial state j and by v_j the variable representing the value at state j. Then the optimal value function magnitude for state j, v_j^*, is the solution to the following linear programming formulation:

$$\max \sum_{j\in\mathcal{S}} \gamma_j v_j \tag{5.9}$$

subject to

$$v_i - \lambda \sum_{j\in\mathcal{S}} \mathcal{P}_{\pi_\alpha}(i,j)v_j \leq \mathcal{C}_i(\pi_\alpha), \qquad \pi_\alpha \in \mathcal{A}', i \in \mathcal{S} \tag{5.10}$$

$$v_i \geq 0, \quad i \in \mathcal{S}. \tag{5.11}$$

Let $\omega(i, a_\alpha)$ be the dual decision variable corresponding to (5.10), then the optimal policy can be derived from the solution of the following linear program, which is the dual of (5.9)-(5.11):

$$\min \sum_{i\in\mathcal{S}} \sum_{\pi_\alpha\in\mathcal{A}'} \mathcal{C}_i(\pi_\alpha)\omega(i,\pi_\alpha) \tag{5.12}$$

subject to

$$\sum_{\pi_\alpha \in \mathcal{A}'} \omega(j, \pi_\alpha) - \lambda \sum_{i \in \mathcal{S}} \sum_{\pi_\alpha \in \mathcal{A}'} \mathcal{P}_{\pi_\alpha}(i, j) \omega(i, \pi_\alpha) \geq \gamma_j, \; j \in \mathcal{S}, \tag{5.13}$$

$$\omega(i, \pi_\alpha) \geq 0, \; i \in \mathcal{S}, \pi_\alpha \in \mathcal{A}'. \tag{5.14}$$

The optimal solution $\omega^*(i, \pi_\alpha)$ is the fraction of periods during which facility is in state i and action π_α is taken. [48, p.228] shows that for any positive choice of $\gamma_j, \; j \in \mathcal{S}$, the resulting linear programs have the same optimal basis. Moreover, the dual optimal solution is equivalent to the optimal deterministic policy for the discounted MDP as follows. For every $i \in \mathcal{S}$ and $\pi_\alpha \in \mathcal{A}'$, there exist only one positive $\omega(i, \pi_\alpha)$ in the optimal solution of (5.12)-(5.13), meaning there is only one optimal action for each state in the optimal policy π^*_α.

The accuracy of the solution generated by the model highly depends on the accuracy of the deterioration process description. To ensure the accuracy and precision of the input parameters like transitional probabilities representing the deterioration model, a feedback control mechanism as in [39] can be applied to update the deterioration model or model parameter(s) from the realized value of random variables obtained through inspection.

4. Short-term Model Formulation

While facilities can be in a good condition in the steady state, in the short term, there may be facilities with a poor condition that require intensive M&R. The resource required to apply these actions in most cases exceeds the available annual or short-term budget. In such cases, the transportation agency has to make short-term resource allocation decisions. Either resources should be assigned fully to some facilities while M&R actions is deprived to others, or resources should be distributed among facilities to attain a lower quality variation across all facilities in the network. The short-term models presented here are based on the linear expression of CVaR as a function of the decision variables. In particular, in case of the risk function based on pavement roughness deterioration, we will establish such linear dependence analytically (see section 4). In case the linear dependence cannot be established analytically, a linear approximation of the CVaR can be used as follows. First, the CVaR level that can be achieved under each action (or M&R intensity) is calculated from the corresponding distribution. Then, CVaR approximation function can be obtained from linear or polynomial fitting of these discrete points. Figure 15 depicts such a procedure. Each distribution is the density of facility's condition at the end of planning horizon resulting from applying different intensity M&R actions. The linear approximation of CVaR is obtained from the CVaR values at each distribution.

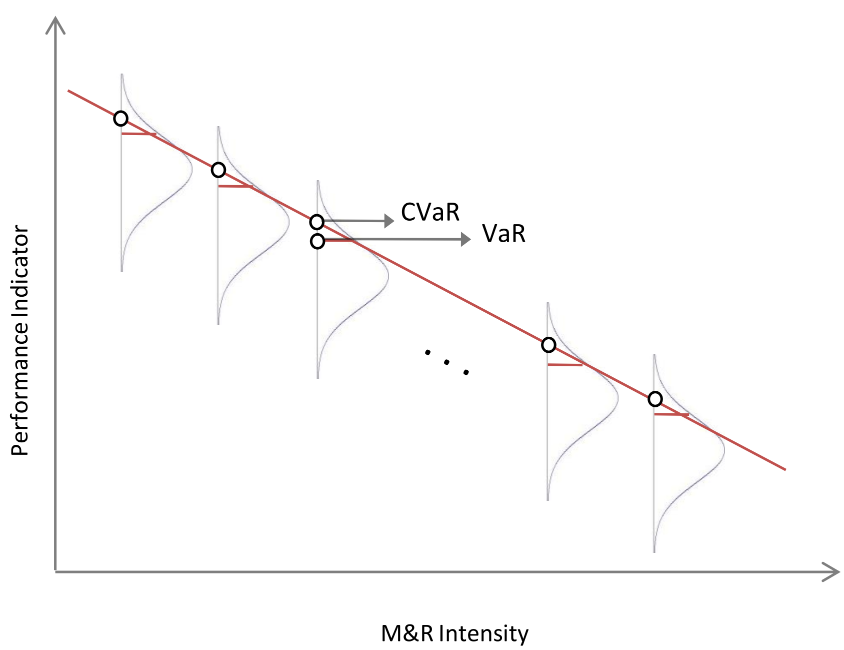


Fig. 15. CVaR approximation.

Denote by $\mathcal{L}_m(x_m)$ the linear approximation of the short-term CVaR for facility m, where x_m is the budget allocated to facility m. Let B_T be the annual budget available to the agency and LB_m and UB_m be the lower and upper bound of budget allocated to facility m, respectively. Denote by $C_\alpha^m(x_m)$ the α-CVaR corresponding to facility m and by C_α the variable representing the upper bound on the CVaR of each facility in the network. We will use the linear approximation $\mathcal{L}_m(x_m)$ to represent $C_\alpha^m(x_m)$, i.e., we assume that

$$\mathcal{L}_m(x_m) = C_\alpha^m(x_m), \quad m \in F,$$

where $F \subset \{1, \ldots, M\}$ is the subset of facilities in the network that require M&R. The model in (5.15)-(5.15c) below is the minimax model formulation to minimize the largest CVaR among all facilities in the network. We call this model MIN-MAX-CVaR.

$$\min C_\alpha \tag{5.15}$$

subject to

$$\mathcal{L}_m(x_m) \leq C_\alpha, \quad m \in F, \tag{5.16}$$

$$\sum_{m \in F} x_m \leq \ B_T, \tag{5.17}$$

$$LB_m \leq x_m \leq UB_m, \quad m \in F. \tag{5.18}$$

Here constraint (5.15a) provides an upper bound on the highest CVaR of a facility in the network, (5.15b) limits the total budget by B_T, and constraint (5.15c) bounds allocation to each facility within the lower and upper bound limits.

The second, alternative model written in (5.19)-(5.19b) below has the objective of minimizing the sum of CVaR over all facilities. We call this model MIN-SUM-CVaR:

$$\min \sum_{m \in F} \mathcal{L}_m(x_m) \tag{5.19}$$

subject to

$$\sum_{i \in F} x_m \leq B_T, \tag{5.20}$$

$$LB_m \leq x_m \leq UB_m, \quad m \in F. \tag{5.21}$$

The constraints in this model are the same as (5.17)-(5.18). Next, the general models presented in this section are illustrated for a specific setting.

D. Network Rehabilitation Models with Risk Defined by Pavement Deterioration

To apply the proposed methodology for network rehabilitation decisions, we will utilize the stochastic deterioration model (5.1) in combination with the continuous pavement state model proposed by Ouyang and Madanat [46] for overlay and roughness improvement. The possible actions will be associated with the thickness of pavement overlay applied to the facility. The risk in this case will be defined as the facility's pavement roughness, and for a fixed overlay thickness the corresponding risk function will be given by the stochastic roughness deterioration model (5.1). Therefore, the risk function will have a lognormal distribution for each action. As will be shown below, the risk function's CVaR will depend linearly on the decision variable w_m representing the overlay thickness. This will allow us to write linear programming formulations for both MIN-MAX-CVaR and MIN-SUM-CVaR models. We describe the model in detail in the following subsections.

1. Continuous Pavement State Model

The following model was proposed by Ouyang and madanat [46] for overlay and roughness improvement. Let $G(w_m, s_m^0)$ denote the roughness improvement after applying w_m mm of overlay on the pavement of facility m with s_m^0 as the initial roughness, then

$$G(w_m, s_m^0) = \frac{g_1 w_m}{g_2 + g_3/s_m^0}, \tag{5.22}$$

$$\text{where } \; w_m \leq g_2 s_m^0 + g_3, \tag{5.23}$$

$$g_1 = 0.66, \; g_2 = 0.55, \; g_3 = 18.3.$$

The corresponding agency cost function is given by

$$M(w_m) = m_1 w_m + m_2, \tag{5.24}$$

$$m_1 = 3,000, \; m_2 = 150,000.$$

Given the initial roughness s_m^0, this model can be used to relate the desired roughness s_m^1 of the facility after applying the overlay to the thickness of overlay. Namely, we obtain the following linear relationship:

$$s_m^1 = s_m^0 - \frac{g_1 w_m}{g_2 + g_3/s_m^0}. \tag{5.25}$$

2. The Risk Function and Its CVaR

We will define the risk function $f_m(s_m^1, \xi_m)$ for facility m according to the stochastic deterioration model (5.1) as follows:

$$f_m(s_m^1, \xi_m) = s_m^1 \exp(\xi_m \tau),$$

where s_m^1 represents the roughness after M&R action (which we treat as the initial condition in this case) and $\xi_m \sim N(\mu_m, \sigma_m^2)$. Hence $\eta_m = f_m(s_m^1, \xi_m)/s_m^1$ has the lognormal probability density function

$$p_{\eta_m}(y) = \frac{1}{y\tau\sigma_m\sqrt{2\pi}} \exp\left(-\frac{(\ln(y) - \tau\mu_m)^2}{2(\tau\sigma_m)^2}\right). \tag{5.26}$$

Let

$$F_{\eta_m}(y) = \frac{1}{2} + \frac{1}{2}\mathrm{erf}\left(\frac{\ln(y) - \mu_m\tau}{\sigma_m\tau\sqrt{2}}\right)$$

be the cumulative distribution function of η_m, and let ζ_α and ϕ_α be the α-VaR and α-CVaR for η_m, respectively. We have

$$\zeta_\alpha = F_{\eta_m}^{-1}(\alpha) = \exp(\tau\mu_m + \sqrt{2}\tau\sigma_m \mathrm{erf}^{-1}(2\alpha - 1)). \tag{5.27}$$

Using the definition (5.5) of CVaR and equation (5.27) for ζ_α, we obtain:

$$\begin{aligned} \phi_\alpha &= \tfrac{1}{(1-\alpha)} \tfrac{1}{\tau\sigma_m\sqrt{2\pi}} \int_{\zeta_\alpha(x_m)}^{\infty} \exp\left(-\tfrac{(\ln(y)-\tau\mu_m)^2}{2(\tau_m\sigma_m)^2}\right) dy \\ &= \tfrac{1}{(1-\alpha)} \exp\left(\tau\mu_m + \tfrac{(\tau\sigma_m)^2}{2}\right) \Phi\left(\tfrac{-\ln(\zeta_\alpha(x_m))+\tau\mu_m+(\tau\sigma_m)^2}{\tau\sigma_m}\right) \\ &= \tfrac{1}{(1-\alpha)} \exp\left(\tau\mu_m + \tfrac{(\tau\sigma_m)^2}{2}\right) \Phi\left(-\sqrt{2}\mathrm{erf}^{-1}(2\alpha - 1) + \tau\sigma_m\right), \end{aligned} \tag{5.28}$$

where $\Phi(\cdot)$ is the cumulative distribution function of the standard normal distribution. Since $f_m(s_m^1, \xi_m) = \eta_m s_m^1$, the α-CVaR $\phi_\alpha^f(s_m^1)$ for the loss function f_m is calculated by the following equation:

$$\phi_\alpha^f(s_m^1) = s_m^1\phi_\alpha(x_m) = \frac{s_m^1}{(1-\alpha)} \exp\left(\tau\mu_m + \frac{(\tau\sigma_m)^2}{2}\right) \Phi\left(-\sqrt{2}\mathrm{erf}^{-1}(2\alpha - 1) + \tau\sigma_m\right). \tag{5.29}$$

This formula can be used to calculate the required initial condition to guarantee a performance measure for pavement performance (e.g., roughness) under certain level of risk. For example, if $\xi_m \sim N(.05, .01)$ and $\tau = 10$, to achieve 95%-CVaR = 50 (lower number represents a better roughness level), the initial condition resulting from

M&R action should be $s_m^1 \leq \frac{50}{2.028} = 24.65$.

3. Short-term Models

Let us introduce the following notation:

$$k(\mu_i, \sigma_i, \tau, \alpha) = \frac{1}{(1-\alpha)} \exp\left(\tau\mu_i + \frac{(\tau\sigma_i)^2}{2}\right) \Phi\left(-\sqrt{2}\text{erf}^{-1}(2\alpha - 1) + \tau\sigma_i\right)$$

Note that $k(\mu_i, \sigma_i, \tau, \alpha)$ does not depend on the decision variables, which are given by the overlay thickness $w_i, i \in F$ for each facility and the largest α-CVaR C_α of a facility in the network. Summarizing the equations in the previous subsections, we obtain the following LP model for MIN-MAX-CVaR:

$$\min C_\alpha \tag{5.30}$$

subject to

$$\left(s_m^0 - \frac{0.66 w_m}{0.55 + 18.3/s_m^0}\right) k(\mu_m, \sigma_m, \tau, \alpha) \leq C_\alpha, \quad m \in F, \tag{5.31}$$

$$\sum_{m \in F} (3,000 w_m + 150,000) \leq \; B_T, \tag{5.32}$$

$$10 \leq w_m \leq 0.55 s_m^0 + 18.3. \tag{5.33}$$

Similarly, the MIN-SUM-CVaR model is given by:

$$\min \sum_{i \in F} \left(s_m^0 - \frac{0.66 w_m}{0.55 + 18.3/s_m^0}\right) k(\mu_m, \sigma_m, \tau, \alpha) \tag{5.34}$$

subject to

$$\sum_{m \in F} (3,000 w_m + 150,000) \leq \; B_T, \tag{5.35}$$

$$10 \leq w_m \leq 0.55 s_m^0 + 18.3. \tag{5.36}$$

E. Numerical Study

We consider a network of 20 facilities with different initial conditions for the numerical study. The network is depicted in Figure 16. The number on each link represents the initial roughness of the facility. We first derive the long-term steady-state policy for facilities that are within the state-space with no budget restriction and then use short-term models with budget restriction assuming all facilities require rehabilitation action. Note that there are facilities with failed condition in the network (their condition is outside of the state-space and require maximum rehabilitation) that we would like to rehabilitate along with facilities in normal condition. The deterioration parameter ξ_m is assumed to be a random variable normally distributed with $\mu_m = .05$ and $\sigma_m = .01$ for all facilities $m = 1, \ldots, 20$. First, we obtain the long-term risk-averse rehabilitation policies that satisfy the 90%-CVaR level of 45 QI (QI is the index of pavement roughness). The state-space is a grid of 10 discrete points in the interval $[5, 50]$. For each roughness level, the distribution of deterioration rate and, hence, distribution of roughness in the 10-year planning horizon is constructed by using the formulas in (5.2). Figure 17 illustrates the distribution of facility's deterioration rate s/s^0 with $\mu = .05$ and $\sigma = .01$ for the 10 year time horizons relative to 1 and 20 year time horizons. As Figure 17 shows, the deterioration rates widen for longer time frames. The increase in deterioration rate variance highlights the importance of risk management for M&R actions with longer time frame.

1. Long-term Model Solution

For numerical computations, three actions are considered in the action set. The actions are assumed to be the minimum (min), medium (mid) and maximum (max) intensity rehabilitation. The minimum rehabilitation action is the one that satisfies

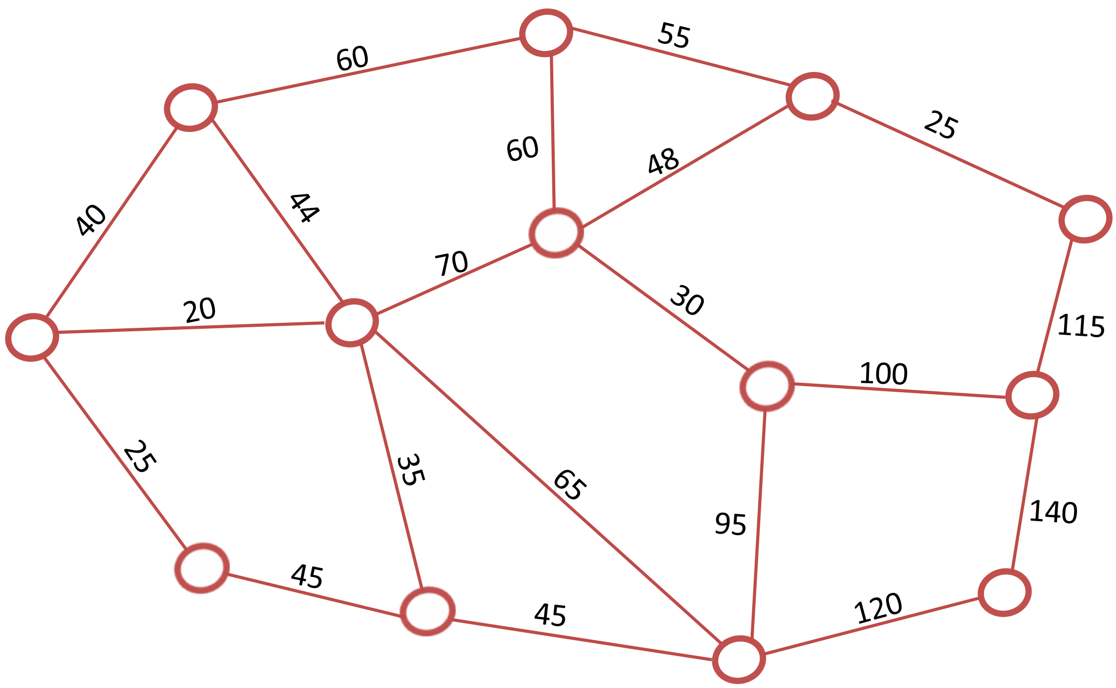


Fig. 16. Network of pavement facilities.

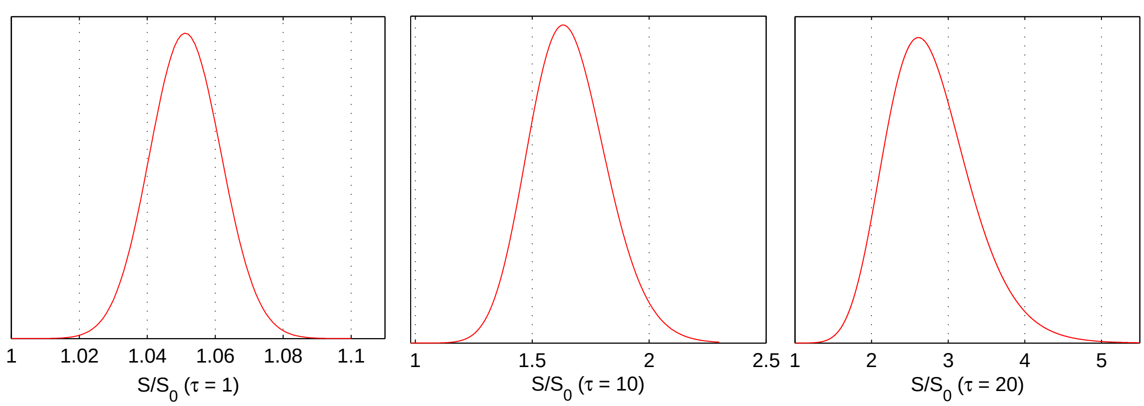


Fig. 17. Deterioration distributions.

CVaR with a minimum cost and, depending on the initial state, will be different; the maximum rehabilitation action is the maximum rehabilitation possible, and the medium rehabilitation is considered to be the average of min and max actions. The transitional probabilities for each of these actions along with the cost of actions at each state are generated. The transitional probabilities can be found in Table XIII in Appendix A.

Table XI shows the cost of action in each state and optimal long-term risk-averse rehabilitation actions, respectively, that are obtained by solving the long-term model (5.12)-(5.14). The cost of action is different for each state since the minimum rehabilitation intensity to satisfy the targeted risk level will be different (zero cost indicates no action). The solution yields the expected required budget of \$144339.7 for the planning horizon. This solution only applies to facilities with initial condition within the $[5, 50]$ range. The results show an increase in rehabilitation intensity with increase in initial condition. Maximum rehabilitation rolls back the facility to the state that requires no action to achieve the targeted risk level (lower number represents a better roughness level).

Table XI. Cost of action and optimal steady-state solution

Action/State	5	10	15	20	25	30	35	40	45	50
Min	0	0	0	0	165000	189000	210000	231000	249000	264000
Mid	183000	186000	190500	195000	207000	222000	237000	252000	265500	276000
Max	216000	222000	231000	240000	249000	255000	264000	273000	282000	288000
Optimal Solution	Min	Min	Min	Min	Max	Max	Max	Max	Max	Max

2. Short-term Models Solution

The time frame assumed for short-term CVaR approximation is 2-year. Table XII summarizes the comparison of the results that each short-term model generated on the test network of 20 pavement links with various initial conditions. The budget is limited to $4.5 million, which is below the required budget to maintain the network at 45 QI. The table shows the initial condition of each facility along with CVaR resulting from applying the solution obtained from the two models. In the MIN-SUM-CVaR solution, there are few facilities with a very poor quality, while in the MIN-MAX-CVaR solution all facilities' quality is bounded by 59.72, which results in a slightly higher total sum of CVaR. Comparison of the standard deviations of the two solutions suggests a much better distribution of facility quality in the MIN-MAX-CVaR solution.

The results from two short-term models show that narrowing the quality gap or reducing the CVaR variance can provide more consistency in pavement condition across the network with the slightly lower total quality level. Comparing the results from MIN-MAX-CVaR and MIN-SUM-CVaR models clearly indicates the advantage of the MIN-MAX-CVaR model, since the small improvement in total network quality that the MIN-SUM-CVaR model offers does not justify the high variance in the facility's quality level across the network. Figure 18 compares the computed CVaR of facilities under the solutions obtained from the two models.

F. Summary and Conclusions

In this chapter, we presented a general methodology for determining optimal risk-based maintenance and rehabilitation (M&R) policies for transportation infrastructure that utilizes the CVaR risk measure. The approach enables network M&R plan-

Table XII. Results of the short-term models

		Min-Sum-CVaR			Min-Max-CVaR		
m	**Initial Condition**	w_m	Cost	CVaR	w_m	Cost	CVaR
1	40	10	180000	38.29	10	180000	38.29
2	25	10	180000	22.72	10	180000	22.72
3	25	10	180000	22.72	10	180000	22.72
4	44	10	180000	42.55	10	180000	42.55
5	25	10	180000	22.72	10	180000	22.72
6	45	10	180000	43.61	10	180000	43.61
7	60	10	180000	59.85	10.1463	180439	59.72
8	70	10	180000	70.82	21.9236	215771	59.72
9	65	10	180000	65.32	16.1674	198502	59.72
10	45	10	180000	43.61	10	180000	43.61
11	60	10	180000	59.85	10.1463	180439	59.72
12	55	10	180000	54.40	10	180000	54.40
13	48	10	180000	46.83	10	180000	46.83
14	30	10	180000	27.83	10	180000	27.83
15	95	15.55	196650	92.93	48.1948	294584	59.72
16	25	10	180000	22.72	10	180000	22.72
17	100	73.3	369900	38.92	53.1228	309368	59.72
18	120	84.3	402900	46.70	72.2002	366601	59.72
19	115	81.55	394650	44.76	67.5095	352528	59.72
20	140	95.3	435900	54.49	90.5889	421767	59.72
SUM			4500000	921.64		4499999	925.48
Standard Deviation				18.28			15.12

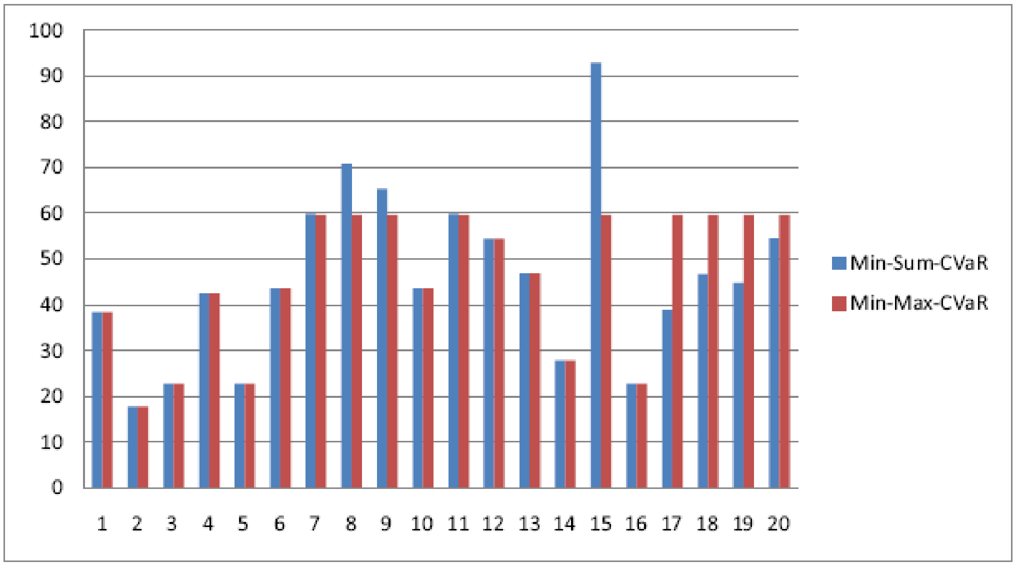


Fig. 18. Expected CVaR after applying short-term solutions.

ners to determine optimal policies that guarantee a certain performance level and can be used effectively in a wide variety of situations, such as ensuring safety and security of certain critical infrastructure facilities. It can also be used as a measure to define a level of service for network users or simply as a means to solicit adequate level of funding from the corresponding authorities. Furthermore, this approach can be used to make network-level short-term resource allocation decisions when resources are not sufficient to implement long-term policies.

The long-term model is constructed in the MDP framework to minimize the cost of network M&R actions such that a certain level for a given performance indicator is guaranteed. The risk-averse actions and transitional probabilities for the MDP are constructed from the probability distribution of facility at the end of planning horizon. Two short-term models that are used to make resource allocation decisions are presented: the MIN-MAX-CVaR model minimizes the highest CVaR over all facilities, and the MIN-SUM-CVaR model minimizes the sum of CVaR of all facilities subject to the budget restrictions.

As an example of application of the proposed methodology, we presented a method of finding risk-averse rehabilitation policies for networks of transportation infrastructure under deterioration uncertainty. The numerical results on short-term rehabilitation decisions from two models show that the first model generates a solution with a lower variance across the network roughness levels, but with slightly higher sum of CVaR for roughness for all facilities. Meanwhile, the second model gives a solution with a high roughness variance among the facilities, but with the sum of CVaR that is smaller compared to the first model. In other words, risk-averse decision makers emphasize fairness in budget allocation by using the MIN-MAX-CVaR model in which all links in the network are considered equally important, while by using the MIN-SUM-CVaR model, decision makers emphasize cost effectiveness.

CHAPTER VI

SUMMARY AND CONCLUSIONS

The goal of this dissertation research is to model risk in delivery, operation and maintenance phases of infrastructure asset management. Infrastructure asset management is a decision-making framework that is founded on economics as well as engineering principles to maintain infrastructure assets in a efficient and reliable way. This dissertation presents a valuation approach for large-scale engineering projects and a framework for risk-based maintenance and rehabilitation resource allocation.

Chapter I discusses the arrangement between private and public entities to own and operate infrastructure assets and the benefits of private sector involvement in asset management. In addition, potential risks in privatizing these assets are described and public-private partnerships, their benefits and common arrangements in transportation infrastructure are discussed. An important part of privatizing infrastructure assets is project valuation process that includes identifying and quantifying project risks. Similar to any other risky asset, the value of an infrastructure project is proportional to its underlying risks and prospective return. Infrastructure projects are capital intensive and require significant investment, but at the same time they cannot be traded in open markets. The large-scale and non-traded nature of infrastructure projects require special consideration in valuing such assets. One of the challenges arising in the valuing risky assets stems from uncertainties that are unique to the project.

Further, in Chapter II, maintenance and rehabilitation of infrastructure facilities are discussed. Infrastructure facilities must undergo a number of M&R actions throughout their life to maximize their service lives. Effective maintenance and rehabilitation planning not only significantly lowers the total life cycle cost of an infrastructure facility, but also provide a consistent level of service for users. Many transportation agencies across the nation have developed systems to manage transportation infrastructure assets in a cost-effective and efficient manner. However, finding optimal network-level M&R policies is challenging because of uncertain deterioration of infrastructure facilities and resource limitations.

Chapter III presents a formal definition of risk along with standard risk management practices. Expected utility and certainty equivalent as traditional measures of risk as well as coherent risk measures are discussed. Definition of coherent risk measures and Value at Risk and Conditional Value at Risk as two widely used measure of downside risks for financial risk management are described. As a convex measure of risk, CVaR can be represented with linear constraints in a convex optimization model.

Chapter IV introduces a valuation approach based on mean-CVaR portfolio optimization in which a risk-averse decision-maker seeks to maximize the expected return subject to downside risk represented by CVaR. We showed that, in complete markets, the value obtained from this approach is equal to the value obtained from standard option pricing. Further, if project's uncertainties are well defined and can be decomposed into market uncertainties and private uncertainties that are represented by a set of subjective probabilities based on firm's past performance or expert opinion, one can use CVaR as a surrogate for private risk and standard option pricing methods for market risk. The risk-neutral probabilities are needed to manage market risks and subjective probabilities along with a α percentile for valuing private risks. We introduce the Coherent Valuation Procedure for valuing risky projects in partially complete markets and show that the breakeven buying price of a risky project is equal to the value obtained from this valuation procedure.

Compared to an exponential utility function, using CVaR for valuing private risk cash flows results in lower degree of subjectivity as the only parameter used in deriving the value is the confidence level. The range of values obtained in this way shows a greater sensitivity to change in decision-maker's risk tolerance represented by confidence level. In addition, CVP captures decision-maker's exposure to risk in a more natural way and without using a utility function.Further, the application of CVP on valuing a transportation infrastructure PPP is presented in Chapter IV.

On the other hand, compared to traditional discounted cash flow analysis that uses risk-adjusted rate of return for discounting cash flows, CVP displays a lower degree of sensitivity to change in discount rate as only risk-free rate is used to discount cash flows.

Chapter V presents a risk-based framework for prescribing long- and short- term resource allocation decisions for infrastructure networks as the uncertainty in the deterioration process will result in uncertain condition of the facilities in the planning period after applying M&R action. In this chapter, a Markov decision process based model with filtered action space to generate long-term M&R actions for each condition state that satisfy a risk threshold is presented. In addition, by using a convex risk measure, we demonstrate easy to implement short-term resource allocation models that can be used to incorporate fairness and cost effectiveness in the decision making process.

The long-term model is constructed in the MDP framework to minimize the cost of network M&R actions such that a certain level for a given performance indicator is guaranteed. The risk-averse actions and transitional probabilities for the MDP are constructed from the probability distribution of a facility at the end of planning horizon. Two short-term models are proposed to generate resource allocation decisions: the MIN-MAX-CVaR model that minimizes the highest CVaR over all facilities, and the MIN-SUM-CVaR model that minimizes the sum of CVaR of all facilities subject to the budget restrictions.

As an example of application of the proposed methodology in Chapter V, we present a method of finding risk-averse rehabilitation policies for networks of transportation infrastructure under deterioration uncertainty. The numerical results on short-term rehabilitation decisions from two models show that the first model generates a solution with a lower variance across the network roughness levels, but with slightly higher sum of CVaR for roughness for all facilities. Meanwhile, the second model gives a solution with a high roughness variance among the facilities, but with the sum of CVaR that is smaller compared to the first model. In other words, risk-averse decision makers emphasize fairness in budget allocation by using the MIN-MAX-CVaR model in which all links in the network are considered equally important, while by using the MIN-SUM-CVaR model, decision makers emphasize cost effectiveness.

A. Future Research Directions

There are several avenues of future research for the problems investigated in this dissertation. As discussed in Chapter IV, due to large-scale and special nature of infrastructure projects, namely, the existence of private risks or project specific risks, placing an appropriate value on these projects is a challenging task. A possible future research direction would be to investigate and compare the application of various risk measures in quantifying such risks.

A natural extension to the results presented in Chapter IV is to generalize the separation results presented for complete markets, to the case of partially complete markets. In partially complete markets, since project's cash flows will not be fully replicated with CVaR replicating portfolio, a rebalancing term is needed for adjusting the portfolio weights at each time period.

In addition, when there is competition among private bidders to enter into a certain partnership, each private party has to submit a proposal based on its private valuation with the complete/incomplete information of other bidder's risk considerations. Determining the optimal bid amount given that the other bidder's risk tolerances are known is another interesting research direction.

Furthermore, the results presented in Chapter V can be extended to multi-period and system-wide resource allocation decision models that include inspection policies. This problem becomes especially challenging in multi-period planning with varying level of resources at each period.

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APPENDIX A

TRANSITIONAL PROBABILITIES

Table XIII. Transitional probabilities for Min, Mid and Max actions

	1	2	3	4	5	6	7	8	9	10
1	0.000	0.973	0.027	0.000	0.000	0.000	0.000	0.000	0.000	0.000
2	0.000	0.000	0.172	0.801	0.027	0.000	0.000	0.000	0.000	0.000
3	0.000	0.000	0.000	0.017	0.526	0.430	0.026	0.000	0.000	0.000
4	0.000	0.000	0.000	0.000	0.003	0.169	0.552	0.249	0.026	0.001
5	0.000	0.000	0.000	0.000	0.000	0.001	0.050	0.331	0.428	0.163
6	0.000	0.000	0.000	0.000	0.000	0.011	0.216	0.494	0.240	0.036
7	0.000	0.000	0.000	0.000	0.000	0.011	0.216	0.494	0.240	0.036
8	0.000	0.000	0.000	0.000	0.000	0.011	0.216	0.494	0.240	0.036
9	0.000	0.000	0.000	0.000	0.000	0.011	0.216	0.494	0.240	0.036
10	0.000	0.000	0.000	0.000	0.000	0.011	0.216	0.494	0.240	0.036

	1	2	3	4	5	6	7	8	9	10
1	0.160	0.840	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
2	0.000	0.160	0.839	0.001	0.000	0.000	0.000	0.000	0.000	0.000
3	0.000	0.000	0.160	0.810	0.030	0.000	0.000	0.000	0.000	0.000
4	0.000	0.000	0.000	0.160	0.732	0.107	0.001	0.000	0.000	0.000
5	0.000	0.000	0.000	0.001	0.159	0.636	0.195	0.009	0.000	0.000
6	0.000	0.000	0.000	0.001	0.192	0.638	0.164	0.006	0.000	0.000
7	0.000	0.000	0.000	0.000	0.085	0.590	0.302	0.023	0.000	0.000
8	0.000	0.000	0.000	0.000	0.032	0.458	0.445	0.062	0.002	0.000
9	0.000	0.000	0.000	0.000	0.011	0.305	0.540	0.136	0.008	0.000
10	0.000	0.000	0.000	0.000	0.003	0.177	0.554	0.241	0.024	0.001

	1	2	3	4	5	6	7	8	9	10
1	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
2	0.126	0.874	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
3	0.000	0.958	0.042	0.000	0.000	0.000	0.000	0.000	0.000	0.000
4	0.000	0.126	0.872	0.002	0.000	0.000	0.000	0.000	0.000	0.000
5	0.000	0.000	0.751	0.248	0.000	0.000	0.000	0.000	0.000	0.000
6	0.000	0.000	0.126	0.832	0.041	0.000	0.000	0.000	0.000	0.000
7	0.000	0.000	0.004	0.572	0.416	0.008	0.000	0.000	0.000	0.000
8	0.000	0.000	0.000	0.126	0.735	0.136	0.002	0.000	0.000	0.000
9	0.000	0.000	0.000	0.010	0.454	0.494	0.041	0.001	0.000	0.000
10	0.000	0.000	0.000	0.000	0.126	0.625	0.235	0.013	0.000	0.000

VITA

Reza Seyedolshohadaie received his Bachelor of Science degree in industrial engineering from Iran University of Science and Technology in 2000. After working for two years in industry, he started his graduate studies at Prairie View A&M University and earned his Master of Science degree in computer information systems in December 2004. He entered the Ph.D. program in industrial and systems engineering at Texas A&M University in 2005 and received his doctoral degree in August 2011. His research interests include decision-making under uncertainty and risk management with application in infrastructure asset management.

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